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Simple solution to the optimal deployment of cooperative nodes in two-dimensional TOA-based and AOA-based localization system

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Abstract

Cellular-based cooperative communication is a promising technique that allows cooperation among mobile devices not only to increase data throughput but also to improve localization services. For a certain number of cooperative nodes, the geometry plays a significant role in enhancing the accuracy of target. In this paper, a simple solution to the deployment of cooperative nodes aiming at the lowest geometric dilution of precision (GDOP) is proposed suitable for both time of arrival-based cooperative localization system and angle of arrival-based cooperative localization system. Inertia dependence factor is suggested to reveal the relationship among the optimal positions of each cooperative node, which extracts the inertia and the recursiveness of deployment. It is shown in simulations that the proposed solution almost achieves the same GDOP as that of global exclusive method but with less complexity.

Keywords: GDOP, Optimal placement, Cooperative node, TOA, AOA

1 Introduction

With the proliferation of mobile communication technology, wireless positioning has become an increasingly important issue and drawn plenty of attention [1]. Over the past several decades, a variety of location-based services emerged in the fields such as emergency medical systems, asset monitoring and tracking, location sensitive billing, fraud protection, fleet management, and even also including intelligent transportation systems [2, 3]. However, global positioning system (GPS) and cellular positioning system (CPS), two representative positioning technologies applied in many scenarios of daily life, still have some drawbacks [4–7]. On the one hand, in outdoor environment, GPS and CPS signals are sensitive to the interference from space weather, transformer substations, geomagnetic radiation, and etc., which leads to the degradation of the accessibility and continuity of GPS and CPS [8–11]. On the other hand, in indoor environment, GPS and CPS face a challenging radio propagation environment, including mainly multi-path effect and none-line-of-sight (NLOS) effect [12–15].

Indoor wireless communication (IWC) technology, represented by ultrasonic (US), wireless sensor networks (WSN), Bluetooth (BT), radio frequency identification (RFID) and ultra wide band (UWB), provides a novel kind of approach to the data interaction and burgeons rapidly in the last decade. Owing to their low cost, an ocean of IWC devices has been placed in indoor environment to supply context-awareness service for the target. Various positioning systems have also been proposed and unfolded satisfactory performance [16–20]. Meanwhile, thanks to the continual miniaturization of the microelectronic technology, cutting-edge cell phones equipped with IWC modules have been committed to remote control, intelligent entrance guard, instant payment, and tag identification via the connection with surrounding IWC devices. In this situation, if we regard all or part of the surrounding IWC devices that could provide spatial characteristic about the cell phone as cooperative partners, a cooperative localization system on the basis of traditional CPS or GPS can be established, which is

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reasonable to expect a higher location precision and a better stability [21–23].

As shown in Fig. 1, a typical cellular-based cooperative localization system consists of five fundamental components including general nodes (GNs), landmark nodes (LNs), cooperative nodes (CNs), positioning targets (PTs), and info-station (IS). The GNs are the base stations (BSs) that enable to capture the external spatial characteristic and provide general CPS service to the PTs. The LNs represent the infrastructures equipped with IWC modules like RFID readers, infrared receivers, Bluetooth receivers and WiFi access points which can interact data with PTs and afford the internal spatial characteristic about the PTs. Exclusively, we term the LNs that participate in the localization of the PTs and serve like the partner of GNs as CNs. The PTs not only limit to the cell phones but also include other intelligent mobile devices equipped with IWC modules as well as CPS. The IS is a server connecting to both the CN and the GN to converge external and internal spatial characteristic about the PTs. Usually, the spatial characteristic is expressed in the form of the time stamp change, phase variation, and amplitude attenuation of the carrier signal. Triangulation, involving measuring principles such as time of arrival (TOA), angle of arrival (AOA), time difference of arrival (TDOA), and phase of arrival (POA), is one of the positioning algorithms that extracts the geometric properties of triangles from the spatial characteristic so as to estimate the target location [24]. Targeting different applications or services, these principles have unique advantages and disadvantages. Hence, using more than one type of principles at the same time could get better performance. In this paper, our attention focuses on TOA and AOA for their extensive applications and convenient implementations.

The process of tracking a single PT in cooperative localization can be briefly described as follows. First, after moving to indoor environments from the outside, the PT begins to broadcast interrogation signal and monitors the reply back from LNs while maintaining the CPS service. Second, taking no account of accidental interference, the interrogation signal would be heard as long as the PT gets into the surveillance region of any LN. Each LN appraises its computational resource such as power dissipation capacity, arithmetic speed, and dynamic link stability. If the computational resource meets the scheduled requirements of cooperative localization, the LN transmits the acknowledgement signal to the PT and prepares for the cooperation. Third, after verifying the whole acknowledgement signals, the PT chooses a part of LNs, labels them as CNs, and sends ranging signal to them. IS discriminates the ranging signal's difference between the transmitter and receiver, andextracts effective indoor spatial characteristic by leveraging measuring principles such as TOA or AOA. Finally, the location of the PT is estimated by converging the indoor spatial characteristic from the CN and the outdoor spatial characteristic from the GN.

Evidently, a larger number of CNs ensure a higher precision of location but at the cost of more computational complexity and more power consumption involved [25]. For a certain number of CNs, the geometry plays a vital part in the performance of the cooperative positioning system. CNs cited in different position may result in different positioning accuracy. Geometric dilution of precision (GDOP), a relatively convenient indicator calculated on the basis of Cramer-Rao bound (CRB), is commonly used to describe the influence of geometry on the relation between the position determination error and the measurement error [26–31]. Low GDOP is a guarantee for all highly accurate wireless location systems. D. J. Torrieri firstly defined GDOP and analysed the performance of the two most important passive location systems for stationary transmitters [26]. N. Levanon examined the position determination in two-dimensional TOA scenarios and proved that the lowest possible GDOP attainable from range or pseudo-range measurements to

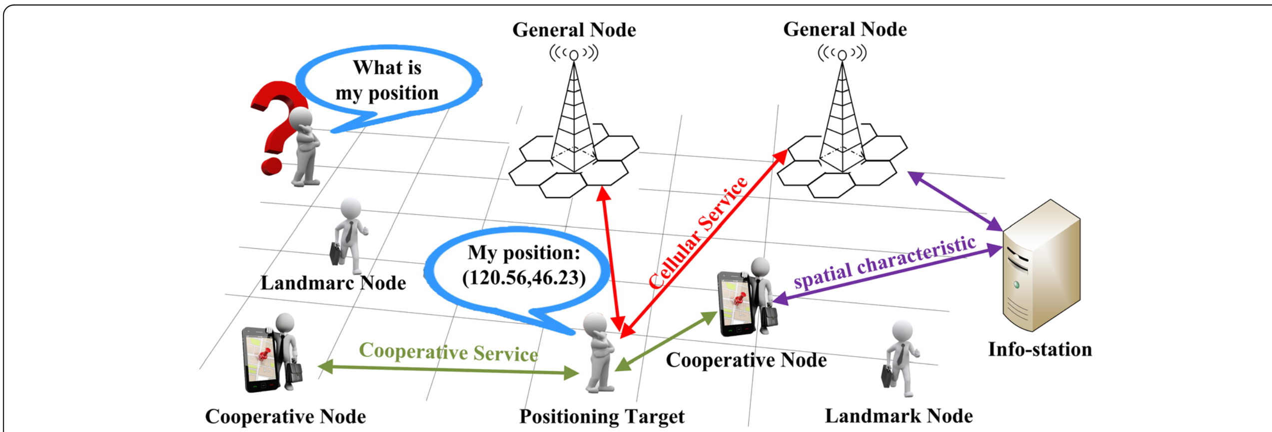

Fig. 1 The components of a typical cellular-based cooperative localization system

N optimally located points is $2/\sqrt{N}$ [27]. A.G. Dempster obtained the expression of GDOP for two-dimensional AOA scenarios [28]. In [29], P. Deng et al. studied the relationship between GDOP and GN number at the centre of a polygon with and without the correlativity between TDOA measurements considered. A GDOP method for computing TDOA and TDOA/AOA under non-line-of-sight environment was also presented. Quan investigated GDOP in absolute range-based wireless location systems with emphasis on its low bounds and introduced angle of coverage as a parameter to describe the geometry between a target and the measuring points [30].

In addition, an efficient deployment scheme of CNs in TOA positioning systems, which aims to minimize the GDOP, is proposed on the basis of generalized eigenvalue decomposition of fisher information matrix (FIM) [31]. In this paper, we termed it as eigenvalue-based approach for ease of illustration. The eigenvalue-based approach, which could obtain optimal performance for one CN and suffer only small loss of performance for more CNs, can dramatically reduce the computational complexity compared with the exhaustive method. Developing a practical arrangement out of the eigenvalue-based approach, however, entails some challenges. First, the eigenvalue-based approach only describes the procedure of scheme rather than theoretically infers the expression of the eigenvalue and eigenvector. Variation of GNs' deployment will trigger the modification of FIM, which may cause recalculation of eigenvalue and eigenvector. Second, the relationship among the optimal position of each CN has not been thoroughly investigated and deeply summarized; unnecessary calculation cannot be avoided when the number of the CNs is huge. Third, the eigenvalue-based approach cannot be directly applied into AOA system since the FIM of AOA is more complicated than that of TOA. To the best of our knowledge, few approaches are reported to tackle the issue mentioned above.

In this paper, we propose another solution to the deployment of CN suitable for both TOA-based cooperative localization system and AOA-based cooperative localization system. Optimal placement mechanism based on partial differentiation, termed as OPMPD, is suggested to expose the optimal position of CN and achieves a more explicit solution. Inertia dependence factor is also introduced to reveal the relationship among the optimal position of each CN, which extracts the inertia and the recursiveness of deployment.

The rest of the paper is organized as follows. CRB based on FIM is reviewed in "Section 2." In "Section 3," we respectively deduce the GDOP expressions of two-dimensional TOA-based cooperative localization system and AOA-based cooperative localization system. Our new solution aiming at the lowest GDOP is detailed in "Section 4." Simulation results are illustrated in "Section 5," which proves the effectiveness and the efficiency of the proposed approach. Finally, the conclusion of our work is presented in "Section 6."

2 Cramer-Rao bound based on FIM

The CRB, which can be derived by the inverse of the FIM, provides the lowest limit on estimation accuracy of any unbiased estimator. Suppose a parameter vector to be estimated is $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots \theta_P]^T$. θ_p stands for the pth unknown parameter, $p \in [1, P]$. P denotes the number of parameters to be estimated. The FIM is then

$$\mathbf{J}(\boldsymbol{\theta}) = -E\left\{\left(\frac{\partial \ln f(\boldsymbol{\Lambda}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)\left(\frac{\partial \ln f(\boldsymbol{\Lambda}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^{\mathrm{T}}\right\} \tag{1}$$

where $\mathbf{J}(\boldsymbol{\theta})$ is a $P \times P$ matrix, $f(\boldsymbol{\Lambda}|\boldsymbol{\theta})$ is the likelihood function of observation vector $\boldsymbol{\Lambda}$, and E represents the expectation operation. Assume that $f(\boldsymbol{\Lambda}|\boldsymbol{\theta})$ follows N dimensional Gaussian distribution denoted as

$$f(\boldsymbol{\Lambda}|\boldsymbol{\theta}) = \frac{1}{\sqrt{(2\pi)^N |\mathbf{Q}|}} e^{-\frac{1}{2}(\boldsymbol{\Lambda}-\boldsymbol{\mu}(\boldsymbol{\theta}))^{\mathrm{T}} \mathbf{Q}^{-1} (\boldsymbol{\Lambda}-\boldsymbol{\mu}(\boldsymbol{\theta}))} \tag{2}$$

where N is the scale of $\boldsymbol{\Lambda}$, $\mathbf{Q}$ is the covariance matrix with $\boldsymbol{\theta}$, $\boldsymbol{\mu}(\boldsymbol{\theta})$ is the expectation of $\boldsymbol{\Lambda}$, and $(\cdot)^T$ and $(\cdot)^{-1}$ respectively represent the transpose and inverse of the matrix. Consider $\mathbf{H} = \frac{\partial \boldsymbol{\mu}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^{\mathrm{T}}}$ as the Jacobian matrix and substitute (2) into (1), $\mathbf{J}(\boldsymbol{\theta})$ will be updated to $\mathbf{J}(\boldsymbol{\theta}) = \mathbf{H}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{H}$, and the CRB of $\boldsymbol{\theta}$ can be expressed as

$$\mathrm{var}\left(\hat{\boldsymbol{\theta}}\right) \geq diag\left(\mathbf{J}^{-1}(\boldsymbol{\theta})\right) = diag\left(\left(\mathbf{H}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{H}\right)^{-1}\right) \tag{3}$$

where var is the variance matrix of unbiased estimators $\hat{\boldsymbol{\theta}} = \left[\hat{\theta}_1, \hat{\theta}_2, \cdots \hat{\theta}_P\right]^T$ and $diag$ denotes the diagonal elements of a matrix.

3 GDOP of two-dimensional localization systems

3.1 GDOP of typical localization system

Consider a two-dimensional typical localization system with a single PT, N GNs. Let $\boldsymbol{\theta} = [x, y]^T$ implies the unknown location of PT and (x_n, y_n) denotes the known location of the nth GN. $n \in [1, N]$. Accordingly, observation vector $\boldsymbol{\Lambda}$ in TOA measurement is $[d_1, d_2, \cdots d_N]^T$ in which d_n denotes the measured distance between the PT and the nth GN given by

$$d_n = ct = \sqrt{(x-x_n)^2 + (y-y_n)^2} = \overline{d_n} + e_n \tag{4}$$

Here, c denotes the light speed. t denotes the absolute arrival time. $\overline{d_n}$ stands for the theoretical value. e_n stands for the measurement error that follows the Gaussian distribution with zero mean and σ_n root-mean-square error. Given the assumption that the measured values in $\mathbf{\Lambda}$ are mutually independent, the cross-covariance matrix can be rewritten as

$$\mathrm{var}\left(\hat{\boldsymbol{\theta}}\right) = \begin{bmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \tag{5}$$

where σ_x^2 and σ_y^2 represent the variance of x and y, respectively.

According to the definition in [26], the GDOP of TOA system can be expressed as

$$GDOP = \frac{\sqrt{\sigma_x^2 + \sigma_y^2}}{\sigma_D^2} \tag{6}$$

with $\sigma_D = \frac{1}{N}\sum_{n=1}^{N}\sigma_n$, where σ_D is the root-mean-square ranging error of system. Suppose $\sigma_1 = \sigma_2 = \sigma_3 = \ldots = \sigma_n = \sigma_r$, (6) can be simplified as $GDOP = \sqrt{G_{11} + G_{22}}$, where G_{11} and G_{22} are the diagonal elements of G given by

$$\boldsymbol{G} = \frac{1}{\sigma_D}\left(\mathbf{H}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{H}\right)^{-1} \tag{7}$$

Considering that all the measurement errors are independent, we have $\mathbf{Q} = \sigma_r^2\mathbf{I}$ with an $N \times N$ identity matrix $\mathbf{I}$. Then the G will become

$$G = \left(H^{\mathrm{T}}H\right)^{-1} \tag{8}$$

Correspondingly, the Jacobian matrix $\mathbf{H}$ of TOA is

$$\mathbf{H}_{\mathrm{TOA}} = \begin{bmatrix} \frac{x-x_1}{d_1} & \frac{y-y_1}{d_1} \\ \frac{x-x_2}{d_2} & \frac{y-y_2}{d_2} \\ \vdots & \vdots \\ \frac{x-x_N}{d_N} & \frac{y-y_N}{d_N} \end{bmatrix} = \begin{bmatrix} -\cos\alpha_1 & -\sin\alpha_1 \\ -\cos\alpha_2 & -\sin\alpha_2 \\ \vdots & \vdots \\ -\cos\alpha_N & -\sin\alpha_N \end{bmatrix} \tag{9}$$

where α_n is the angle of the ray from the PT to the nth GN relative to the positive X-axis.

Substituting (9) into (8), we have

$$\mathbf{G}_{\mathrm{TOA}} = \frac{\begin{bmatrix} \sum_{n=1}^{N}\sin^2\alpha_n & -\sum_{n=1}^{N}\cos\alpha_n\sin\alpha_n \\ -\sum_{n=1}^{N}\cos\alpha_n\sin\alpha_n & \sum_{n=1}^{N}\cos^2\alpha_n \end{bmatrix}}{\sum_{n=1}^{N-1}\sum_{m=n+1}^{N}\sin^2(\alpha_m-\alpha_n)} \tag{10}$$

Therefore, the GDOP of TOA can be derived as

$$GDOP_{\mathrm{TOA}} = \sqrt{\frac{N}{\sum_{n=1}^{N-1}\sum_{m=n+1}^{N}\sin^2(\alpha_m-\alpha_n)}} \tag{11}$$

Similarly, the Jacobian matrix and GDOP of AOA can be obtained with observation vector $\mathbf{\Lambda}$ replaced by $[\alpha_1, \alpha_2, \cdots \alpha_N]^T$

$$\mathbf{H}_{\mathrm{AOA}} = \begin{bmatrix} -\frac{y-y_1}{d_1^2} & \frac{x-x_1}{d_1^2} \\ -\frac{y-y_2}{d_2^2} & \frac{x-x_2}{d_2^2} \\ \vdots & \vdots \\ -\frac{y-y_N}{d_N^2} & \frac{x-x_N}{d_N^2} \end{bmatrix} = \begin{bmatrix} \frac{\sin\alpha_1}{d_1^2} & -\frac{\cos\alpha_1}{d_1^2} \\ \frac{\sin\alpha_2}{d_2^2} & -\frac{\cos\alpha_2}{d_2^2} \\ \vdots & \vdots \\ \frac{\sin\alpha_N}{d_N^2} & -\frac{\cos\alpha_N}{d_N^2} \end{bmatrix} \tag{12}$$

$$GDOP_{\mathrm{AOA}} = \sqrt{\frac{\sum_{n=1}^{N}d_n^{-2}}{\sum_{n=1}^{N-1}\sum_{m=n+1}^{N}d_m^{-2}d_n^{-2}\sin^2(\alpha_m-\alpha_n)}} \tag{13}$$

3.2 GDOP of cooperative localization system

The key idea of cooperative localization systems is improving the accuracy of PTs by introducing more LNs and filtering out CNs from them. Theoretically, the more CN employed, the lower GDOP received. In this section, assume that there are totally M CNs to be allocated to a single PT for reducing the GDOP. After the implementation of assignment, the Jacobian matrixes of the observation vector of TOA-based and AOA-based cooperative localization systems are respectively refreshed to

$$\mathbf{H}_{\text{C-TOA}} = \left[\mathbf{H}_{\mathrm{TOA}}^T \quad \mathbf{H}_{\text{c-TOA}}^T\right]^T \tag{14}$$

$$\mathbf{H}_{\text{C-AOA}} = \left[\mathbf{H}_{\mathrm{AOA}}^T \quad \mathbf{H}_{\text{c-AOA}}^T\right]^T \tag{15}$$

with

$$\mathbf{H}_{\text{C-TOA}} = \begin{bmatrix} \frac{x-x_{c1}}{r_1} & \frac{y-y_{c1}}{r_1} \\ \frac{x-x_{c2}}{r_2} & \frac{y-y_{c2}}{r_2} \\ \vdots & \vdots \\ \frac{x-x_{cM}}{r_M} & \frac{y-y_{cM}}{r_M} \end{bmatrix} = \begin{bmatrix} -\cos\beta_1 & -\sin\beta_1 \\ -\cos\beta_2 & -\sin\beta_2 \\ \vdots & \vdots \\ -\cos\beta_M & -\sin\beta_M \end{bmatrix} \tag{16}$$

$$\mathbf{H}_{\text{C-AOA}} = \begin{bmatrix} -\frac{y-y_{c1}}{r_1^2} & \frac{x-x_{c1}}{r_1^2} \\ -\frac{y-y_{c2}}{r_2^2} & \frac{x-x_{c2}}{r_2^2} \\ \vdots & \vdots \\ -\frac{y-y_{cM}}{r_M^2} & \frac{x-x_{cM}}{r_M^2} \end{bmatrix} = \begin{bmatrix} \frac{\sin\beta_1}{r_1^2} & -\frac{\cos\beta_1}{r_1^2} \\ \frac{\sin\beta_2}{r_2^2} & -\frac{\cos\beta_2}{r_2^2} \\ \vdots & \vdots \\ \frac{\sin\beta_M}{r_M^2} & -\frac{\cos\beta_M}{r_M^2} \end{bmatrix} \tag{17}$$

where β_m is the angle of the ray from the PT to the mth CN relative to the positive X-axis, (x_{cm}, y_{cm}) denotes the coordinate of the mth CN, and r_m denotes the distance between the PT and the mth CN. Geometry of the nodes in two-dimensional cooperative localization scenario is illustrated in Fig. 2.

Hence, the GDOP with respect to M CNs can be calculated as

$$GDOP^{M}_{\text{C-TOA}} = \sqrt{\frac{N+M}{\sum_{n=1}^{N}\sum_{m=1}^{M}\sin^2(\beta_m-\alpha_n) + \sum_{n=1}^{N-1}\sum_{m=n+1}^{N}\sin^2(\alpha_m-\alpha_n) + \sum_{n=1}^{M-1}\sum_{m=n+1}^{M}\sin^2(\beta_m-\beta_n)}} \tag{18}$$

and

$$GDOP^{M}_{\text{C-AOA}} = \sqrt{\frac{\sum_{n=1}^{N} d_n^{-2} + \sum_{m=1}^{M} r_m^{-2}}{\Sigma_1 + \Sigma_2 + \Sigma_3}} \tag{19}$$

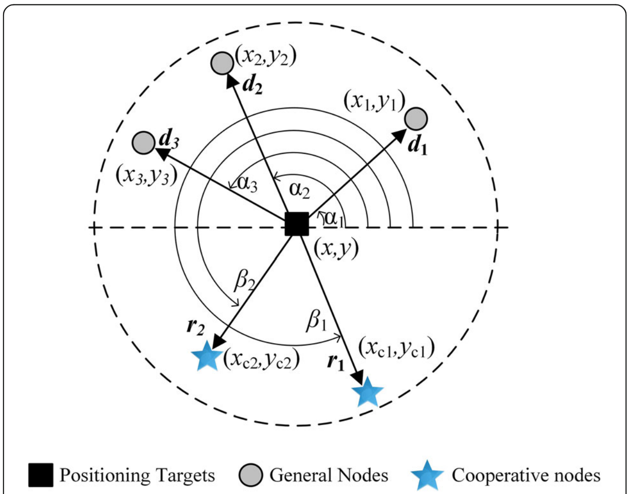

Fig. 2 Geometry of the nodes in two-dimensional cooperative localization scenario

where $\Sigma_1 = \sum_{n=1}^{N}\sum_{m=1}^{M}(d_n r_m)^{-2}\sin^2(\beta_m-\alpha_n)$, $\Sigma_2 = \sum_{n=1}^{M-1}\sum_{m=n+1}^{M}(r_n r_m)^{-2}\sin^2(\beta_m-\beta_n)$, and $\Sigma_3 = \sum_{n=1}^{N-1}\sum_{m=n+1}^{N}(d_n d_m)^{-2}\sin^2(\alpha_m-\alpha_n)$.

4 OPMPD approach

From (18) and (19), we could find that the GDOP of two-dimensional TOA-based cooperative localization systems is affected by the azimuth of the CNs, whereas the GDOP of the AOA-based is relying on both the azimuth and the distance between the PT and the CN. Theoretically, the deployment of cooperative nodes is a multiple parameter optimization problem with high complexity, especially for the large-scale CNs instances. In order to simplify the multiple parameter optimization problem, the OPMPD approach employs stepwise strategy to obtain the optimal position of each CN step by step just as eigenvalue-based approach. Whereas, OPMPD approach has three major advantages. First, OPMPD approach is deduced on the basis of partial differentiation which provides a more explicit scheme. Since there is no need to calculate the generalized eigenvalue and eigenvector of FIM, unnecessary computation is avoided. Second, inertia dependence factor is introduced to indicate the relationship among the optimal position of each CN, which suggests a discipline of the deployment. Third, OPMPD approach is suitable for both two-dimensional TOA-based cooperative localization system and AOA-based cooperative localization system.

4.1 Objective function based on stepwise strategy

In OPMPD approach, the optimal position of the CN to be assigned depends on not only the geometry of the PT and the GN but also the location of the CN already deployed since each CN is placed in sequence. To begin with, assume the first CN shall be placed, the GDOP of the TOA-based cooperative system is found to be

$$GDOP^{1}_{\text{C-TOA}} = \sqrt{\frac{N+1}{\sum_{n=1}^{N}\sin^2(\beta_1-\alpha_n) + \sum_{n=1}^{N-1}\sum_{m=n+1}^{N}\sin^2(\alpha_n-\alpha_m)}} \tag{20}$$

Obviously, the current GDOP is only relevant to β_1 since $\alpha_1, \alpha_2, \ldots, \alpha_N$ is known. Considering that the minimization of GDOP is equivalent to the maximization of $\sum_{n=1}^{N}\sin^2(\beta_1-\alpha_i)$, we extract the component relevant to β_1 and define the initial observation function as

$$Q(\beta_1) = \sum_{n=1}^{N} \sin^2(\beta_1 - \alpha_n) \tag{21}$$

Accordingly, if the former (m−1) CNs have been placed, the observation function with respect to the mth CN will be updated to

$$Q(\beta_m) = \sum_{n=1}^{N} \sin^2(\beta_m - \alpha_n) + \sum_{j=1}^{m-1} \sin^2\left(\beta_m - \beta_{\mathrm{O},j,\mathrm{T}}\right) \tag{22}$$

where $\beta_{\mathrm{O},j,\mathrm{T}}$ denotes the best azimuth of the jth CN that has been acquired.

Similarly, the GDOP of the AOA cooperative system can be expressed as

$$GDOP^1_{\text{C-AOA}} = \sqrt{\frac{r_1^{-2} + \sum_{n=1}^{N} d_n^{-2}}{r_1^{-2}\sum_{n=1}^{N} d_n^{-2}\sin^2(\beta_1 - \alpha_n) + \sum_{n=1}^{N-1}\sum_{m=n+1}^{N} d_n^{-2} d_m^{-2} \sin^2(\alpha_n - \alpha_m)}} \tag{23}$$

Suppose each r_m is a fixed value, then we have

$$Q(\beta_1) = \sum_{n=1}^{N} d_n^{-2} \sin^2(\beta_1 - \alpha_n) \tag{24}$$

$$Q(\beta_m) = \sum_{n=1}^{N} d_n^{-2} \sin^2(\beta_m - \alpha_n) + \sum_{j=1}^{m-1} r_j^{-2} \sin^2\left(\beta_m - \beta_{\mathrm{O},j,\mathrm{T}}\right) \tag{25}$$

In addition, we particularly discuss the effect of r_m in section 4.3 to fully analyze the performance of GDOP of AOA-based cooperative localization system.

4.2 OPMPD approach in TOA-based cooperative localization system

The core concept of OPMPD approach is to get the extremum of $Q(\beta_m)$ by calculating the partial derivative with respect to β_m. Above all, we built the first-order partial differential equations to search the arrest points. For TOA-based cooperative localization systems, let $\partial Q(\beta_1)/\partial \beta_1 = 0$ and define $\beta^1_{1,T}$ and $\beta^2_{1,\mathrm{T}}$ as the arrest points of $Q(\beta_1)$, then we have

$$\begin{cases} \sin 2\beta^1_{1,T} \sum_{n=1}^{N} \cos 2\alpha_n = \cos 2\beta^1_{1,T} \sum_{n=1}^{N} \sin 2\alpha_n \\ \sin 2\beta^2_{1,T} \sum_{n=1}^{N} \cos 2\alpha_n = \cos 2\beta^2_{1,T} \sum_{n=1}^{N} \sin 2\alpha_n \end{cases} \tag{26}$$

Suppose $\sum_{n=1}^{N} \cos 2\alpha_n \neq 0$, $\cos 2\beta^1_{1,T} \neq 0$, and $\cos 2\beta^2_{1,T} \neq 0$, (26) can be refreshed according to the equal ratios theorem

$$\frac{\sin 2\beta^1_{1,T}}{\cos 2\beta^1_{1,T}} = \frac{\sin 2\beta^2_{1,T}}{\cos 2\beta^2_{1,T}} = \frac{\sum_{n=1}^{N} \sin 2\alpha_n}{\sum_{n=1}^{N} \cos 2\alpha_n} \tag{27}$$

Assume $\beta^2_{1,T} > \beta^1_{1,T}$, then

$$\beta^1_{1,T} = \frac{1}{2} \times \arctan\left(\frac{\sum_{n=1}^{N} \sin 2\alpha_n}{\sum_{n=1}^{N} \cos 2\alpha_n}\right) \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \tag{28}$$
$$\beta^2_{1,T} = \beta^1_{1,T} + \frac{1}{2}\pi \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

For easy of depiction and distinction, we term $\beta^1_{1,T}$ and $\beta^2_{1,T}$ in (28) as positive azimuth factor and negative azimuth factor of the first CN, respectively.

Substitute $\beta^1_{1,T}$ and $\beta^2_{1,T}$ into the second-order partial derivative Q'' respectively to find which azimuth factor would obtain the maximum of observation function, then we have

$$Q''\left(\beta^1_{1,T}\right) = 2\left|\vec{A}_{\mathrm{T}}\right|\left|\vec{B}^1_{1,\mathrm{T}}\right| \cos\gamma_1,\ Q''\left(\beta^2_{1,T}\right) = 2\left|\vec{A}_{\mathrm{T}}\right|\left|\vec{B}^2_{1,\mathrm{T}}\right| \cos\gamma_2 \tag{29}$$

with

$$\begin{aligned} \vec{A}_{\mathrm{T}} &= \mathfrak{R} + j\kappa = \sum_{n=1}^{N} \cos 2\alpha_n + j\sum_{n=1}^{N} \sin 2\alpha_n, \\ \vec{B}^1_{1,\mathrm{T}} &= cos2\beta^1_{1,T} + jsin2\beta^1_{1,T}, \\ \vec{B}^2_{1,\mathrm{T}} &= cos2\beta^2_{1,T} + jsin2\beta^2_{1,T} \end{aligned} \tag{30}$$

where $\vec{A}_{\mathrm{T}}$ is the azimuth vector of system, and $\mathfrak{R}$ is the cosine azimuth coefficient. $\vec{B}^1_{1,\mathrm{T}}$ and $\vec{B}^2_{1,\mathrm{T}}$ are defined as the positive azimuth vector and negative azimuth vector of the first CN, j denotes the imaginary unit, γ_1 denotes the angle between $\vec{A}_{\mathrm{T}}$ and $\vec{B}^1_{1,\mathrm{T}}$,and γ_2 denotes the angle between $\vec{A}_{\mathrm{T}}$ and $\vec{B}^2_{1,\mathrm{T}}$.

From (27), (28), (29) and (30), we could draw three useful conclusions. First, the value of $\cos(\gamma_1)$ is either 1 or −1. So does $\cos(\gamma_2)$. Second, the direction of $\vec{B}^1_{1,\mathrm{T}}$ is opposite to that of $\vec{B}^2_{1,\mathrm{T}}$. Third, if $\mathfrak{R} < 0$, the azimuth of $\vec{A}_{\mathrm{T}}$ is confined from $\pi/2$ to $3\pi/2$, thereby leading to $GDOP_{\mathrm{TOA}}\left(\beta^1_{1,T}\right)$ achieving the minimum value with $\cos\gamma_1 = -1$ and $Q''\left(\beta^1_{1,T}\right) < 0$. Conversely, if $\mathfrak{R} > 0$, the azimuth of $\vec{A}_{\mathrm{T}}$ is confined from $-\pi/2$ to $\pi/2$, which means that $GDOP_{\mathrm{TOA}}\left(\beta^2_{1,\mathrm{T}}\right)$ would get the minimum value with $\cos\gamma_2 = -1$ and $Q''\left(\beta^2_{1,\mathrm{T}}\right) < 0$.

Consequently, a deployment mechanism of the first CN can be summarized as

$$\beta_{O,1,T} = \begin{cases} random \ \ if\, \mathfrak{R} = 0 \\ \beta^1_{1,T} \ \ if\, \mathfrak{R} < 0 \\ \beta^2_{1,T} \ \ if\, \mathfrak{R} > 0 \end{cases} \tag{31}$$

where $\beta_{O,1,T}$ represents the optimal azimuth corresponding to the least GDOP and $\overrightarrow{B_{O,1,T}} = cos2\beta_{O,1,T} + jsin2\beta_{O,1,T}$ represents the optimal azimuth vector of the first CN.

After the first CN is arranged at the optimum position, both the azimuth vector of system and the objective function will be refreshed to

$$\overrightarrow{A_T} = \mathfrak{R} + j\kappa = \left(cos2\beta_{O,1,T} + \sum\nolimits_{n=1}^{N}\cos2\alpha_n\right) + j\left(\sin2\beta_{O,1,T} + \sum\nolimits_{n=1}^{N}\sin2\alpha_n\right) \tag{32}$$

$$Q(\beta_2) = \sum\nolimits_{n=1}^{N}\sin^2(\beta_2 - \alpha_n) + \sin^2\left(\beta_2 - \beta_{O,1,T}\right) \tag{33}$$

Define $\beta^1_{2,T}$ and $\beta^2_{2,T}$ as the arrest points of $Q(\beta_2)$, then we have

$$\beta^1_{2,T} = \beta^1_{1,T}, \quad \beta^2_{2,T} = \beta^2_{1,T} \tag{34}$$

since

$$\frac{sin2\beta^1_{2,T}}{\cos2\beta^1_{2,T}} = \frac{\sin2\beta^2_{2,T}}{\cos2\beta^2_{2,T}} = \frac{\sum\nolimits_{n=1}^{N}\sin2\alpha_n + \sin2\beta_{O,1,T}}{\sum\nolimits_{n=1}^{N}\cos2\alpha_n + \cos2\beta_{O,1,T}} = \frac{\sum\nolimits_{n=1}^{N}\sin2\alpha_n}{\sum\nolimits_{n=1}^{N}\cos2\alpha_n} \tag{35}$$

The optimal azimuth of the second CN can be determined as $\mathfrak{R}$ updated to $\sum\nolimits_{n=1}^{N}\cos2\alpha_n + \cos2\beta_{O,1,T}$. From (35), it is clear that either the positive azimuth factor or the negative azimuth factor of each successive CN equals to that of the first CN. Hence, we define β^1_T and β^2_T as the positive azimuth factor and negative azimuth factor of the system where $\beta^1_T = \beta^1_{1,T}$, $\beta^2_T = \beta^2_{1,T}$.

Suppose the absolute value of initial cosine azimuth coefficient is larger than 2, the absolute value of cosine azimuth coefficient refreshed will decrease as the CN is introduced since the directions of $\overrightarrow{B_{O,1,T}}$ and $\overrightarrow{B_{O,2,T}}$ are opposite to that of initial $\overrightarrow{A_T}$. That is

$$\left|\cos2\beta_{O,1,T} + \cos2\beta_{O,2,T} + \sum_{n=1}^{N}\cos2\alpha_n\right| < \left|\cos2\beta_{O,1,T} + \sum_{n=1}^{N}\cos2\alpha_n\right| < \left|\sum_{n=1}^{N}\cos2\alpha_n\right| \tag{36}$$

Moreover, if initial cosine azimuth coefficient is even greater, there exists a theoretical quantity of the CNs termed as Z_T satisfying

$$\beta_{O,Z_T+1,T} \neq \beta_{O,Z_T,T} \cdots = \beta_{O,2,T} = \beta_{O,1,T} \tag{37}$$

which means that the optimal azimuth of the former Z_T CNs equal to $\beta_{O,1,T}$ rather than $\beta_{O,Z_T+1,T}$.

For simplicity, we define Z_T as inertia dependency factor and estimate it by

$$\begin{cases} \left((Z_T - 1)\cdot\cos2\beta_{O,1,T} + \sum_{n=1}^{N}\cos2\alpha_n\right)\sum_{n=1}^{N}\cos2\alpha_n > 0 \\ \left(Z_T\cos2\beta_{O,1,T} + \sum_{n=1}^{N}\cos2\alpha_n\right)\sum_{n=1}^{N}\cos2\alpha_n < 0 \end{cases} \tag{38}$$

From (38), we have

$$\begin{cases} 1 + (Z_T - 1)V > 0 \\ Z_T V + 1 < 0 \end{cases} \tag{39}$$

with

$$V = \frac{\sin2\beta_{O,1,T}}{\sum_{n=1}^{N} sin2\alpha_n} = \frac{\cos2\beta_{O,1,T}}{\sum_{n=1}^{N}\cos2\alpha_n} \tag{40}$$

Hence,

$$Z_T = \sqrt{\left(\sum_{n=1}^{N}\sin2\alpha_n\right)^2 + \left(\sum_{n=1}^{N}\cos2\alpha_n\right)^2} \tag{41}$$

where ⌈⌉ denotes of the operation of rounding up.

The effect of the magnitude of M on the deployment is also taken into consideration in our research. Assume the initial value of $\mathfrak{R}$ is less than zero. If $Z_T \geq M$, the relationship among the optimal azimuth of each CN is found to be

$$\beta_{O,M,T} \cdots = \beta_{O,2,T} = \beta_{O,1,T} = \beta^1_T \tag{42}$$

On the contrary, if $Z_T < M$, it is necessary to explore the optimal azimuth of CN whose index is larger than Z_T. For the $(Z_T + 1)$th CN to be placed, $\mathfrak{R}$ is updated to

$$\Re = Z_{\mathrm{T}} \cdot \cos 2\beta_{\mathrm{T}}^{1} + \sum_{n=1}^{N} \cos 2\alpha_n \tag{43}$$

Since $\left(Z_{\mathrm{T}} \cos 2\beta_{\mathrm{T}}^{1} + \sum_{n=1}^{N} \cos 2\alpha_n \right) < 0$, we have

$$\beta_{\mathrm{O},Z_{\mathrm{T}}+1,\mathrm{T}} = \beta_{\mathrm{T}}^{2} \tag{44}$$

Then, for the $(Z_{\mathrm{T}} + 2)$th CN to be placed, we have

$$\begin{aligned}\Re &= Z_{\mathrm{T}} \cdot \cos 2\beta_{\mathrm{T}}^{1} + \sum_{n=1}^{N} \cos 2\alpha_n \\ &\quad + \cos 2\beta_{\mathrm{O},Z_{\mathrm{T}}+1,\mathrm{T}} \\ &= (Z_{\mathrm{T}}-1) \cdot \cos 2\beta_{\mathrm{T}}^{1} + \sum_{n=1}^{N} \cos 2\alpha_n \end{aligned} \tag{45}$$

So

$$\beta_{\mathrm{O},Z_{\mathrm{T}}+2,\mathrm{T}} = \beta_{\mathrm{T}}^{1} \tag{46}$$

Consequently, the relationship among the optimal azimuth of each CN is found to be

$$\begin{aligned}\beta_{\mathrm{O},Z_{\mathrm{T}}+2K,\mathrm{T}} &= \ldots = \beta_{\mathrm{O},Z_{\mathrm{T}}+4,\mathrm{T}} = \beta_{\mathrm{O},Z_{\mathrm{T}}+2,\mathrm{T}} \\ &= \beta_{\mathrm{O},Z_{\mathrm{T}},\mathrm{T}} \cdots = \beta_{\mathrm{O},2,\mathrm{T}} = \beta_{\mathrm{O},1,\mathrm{T}} \\ &= \beta_{\mathrm{T}}^{1} \beta_{\mathrm{O},Z_{\mathrm{T}}+2K-1,\mathrm{T}} = \ldots = \beta_{\mathrm{O},Z_{\mathrm{T}}+3,\mathrm{T}} \\ &= \beta_{\mathrm{O},Z_{\mathrm{T}}+1,\mathrm{T}} = \beta_{\mathrm{T}}^{2} \end{aligned} \tag{47}$$

with

$$K = \lfloor 0.5(M + 1 - Z_{\mathrm{T}}) \rfloor \tag{48}$$

where $\lfloor\,\rfloor$ denotes of the coperation of rounding down. Similar conclusion can also be derived in the case of the initial value of $\Re$ equal or greater than zero.

Hence, a solution to the optimal deployment of cooperative nodes in two-dimensional TOA-based localization system aiming at the lowest GDOP can be summarized as follows.

Step 1: According to the spatial relationship between the GN and PT, establish the initial observation function and calculate β_{T}^{1}, β_{T}^{2}, Z_{T} and the initial $\Re$.

Step 2: Determine the optimal azimuth of the first CN from β_{T}^{1} and β_{T}^{2}, according to the polarity of $\Re$.

Step 3: If $Z_{\mathrm{T}} < M$, deploy the whole CNs in the same azimuth as the first CN assigned.

Step 4: If $Z_{\mathrm{T}} \geq M$, deploy the former Z_{T} CNs in the same azimuth as the first CN assigned while place the $(Z_{\mathrm{T}}+1)$th CN in the azimuth vertical to that of the first CN. Then, β_{T}^{1} and β_{T}^{2} are alternatively chosen as the optimal azimuth of the rest CNs.

4.3 OPMPD approach in AOA-based cooperative localization system

As mentioned in the initial portion of the "Section 4," in two-dimensional AOA-based localization system, both the azimuth and distance of each CN exert an influence on the performance of the GDOP.

First of all, we analyze the influence of β_1 on the value of $GDOP_{\text{C-AOA}}^{1}$ depicted in (23) assuming each r_m is a fixed value. With the help of a method similar to that in TOA-based system, the positive azimuth factor and negative azimuth factor of the system can be derived as

$$\begin{aligned}\beta_{\mathrm{A}}^{1} &= \beta_{1,\mathrm{A}}^{1} = 0.5 \times \arctan\left(\sum\nolimits_{n=1}^{N} d_n^{-2} \sin 2\alpha_n \Big/ \sum\nolimits_{n=1}^{N} d_n^{-2} \cos 2\alpha_n \right) \\ \beta_{\mathrm{A}}^{2} &= \beta_{1,\mathrm{A}}^{2} = \beta_{1,\mathrm{A}}^{1} + 0.5\pi \end{aligned} \tag{49}$$

where $\beta_{1,\mathrm{A}}^{1}$ and $\beta_{1,\mathrm{A}}^{2}$ denote the arrest points of the observation function depicted in (24). The azimuth vector of system can also be obtained by

$$\vec{A}_{\mathrm{A}} = \Re + j\kappa = \sum\nolimits_{n=1}^{N} d_n^{-2} \cos 2\alpha_n + j \sum\nolimits_{n=1}^{N} d_n^{-2} \sin 2\alpha_n \tag{50}$$

Accordingly, the optimal azimuth of the first CN is updated to

$$\beta_{\mathrm{O},1,\mathrm{A}} = \begin{cases} random & if\ \Re = 0 \\ \beta_{\mathrm{A}}^{1} & if\ \Re < 0 \\ \beta_{\mathrm{A}}^{2} & if\ \Re > 0 \end{cases} \tag{51}$$

where $\beta_{\mathrm{O},1,\mathrm{A}}$ represents the optimal azimuth corresponding to the least GDOP.

Secondly, we further discuss the influence of r_1 on $GDOP_{\mathrm{AOA}}^{1}$. Suppose the scope of r_m follows

$$r_{1,\min} \le r_m \le r_{1,\max} \tag{52}$$

where $r_{1,\min}$ and $r_{1,\max}$ denote the minimum and maximum. For simplicity, let

$$\begin{aligned} C &= r_1^{-2}, D \\ &= \sum_{n=1}^{N} d_n^{-2}, E \\ &= \sum_{n=1}^{N} d_n^{-2} \sin^2(\beta_1 - \alpha_n), F \\ &= \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} d_n^{-2} d_m^{-2} \sin^2(\alpha_n - \alpha_m) \end{aligned} \tag{53}$$

The first-order derivative of $GDOP_{\mathrm{AOA}}^{1}$ with respect to r_1 is expressed as follows:

$$\frac{\partial GDOP_{\mathrm{AOA}}^{1}(r_1)}{\partial r_1}=\frac{1}{2GDOP_{\mathrm{AOA}}^{1}}\frac{-2(F-DE)}{r_1^3(F+CE)^2} \tag{54}$$

Here, we pay our attention to the polarity of $(F-DE)$ to investigate the monotone property of $GDOP_{\mathrm{AOA}}^{1}$. Based on (53), we have

$$\begin{aligned}2E-D &= -\left(\cos 2\beta\sum_{n=1}^{N}\cos 2\alpha_n d_n^{-2}+\sin 2\beta\sum_{n=1}^{N}\sin 2\alpha_n d_n^{-2}\right)\\ &= \sqrt{\left(\sum_{n=1}^{N}d_n^{-4}\right)+2\sum_{n=1}^{N-1}\sum_{m=i+1}^{N}d_n^{-2}d_m^{-2}\left(1-2sin^2(\alpha_n-\alpha_m)\right)}\end{aligned} \tag{55}$$

$$\begin{aligned}D^2-4F &= \left(\sum_{n=1}^{N}d_n^{-2}\right)^2-4\sum_{n=1}^{N-1}\sum_{m=n+1}^{N}d_n^{-2}d_m^{-2}\sin^2(\alpha_n-\alpha_m)\\ &= \left(\sum_{n=1}^{N}d_n^{-4}\right)+2\sum_{n=1}^{N-1}\sum_{m=n+1}^{N}d_n^{-2}d_m^{-2}\left(1-2sin^2(\alpha_n-\alpha_m)\right)\end{aligned} \tag{56}$$

$$2F-D^2=-\left(\sum_{i=1}^{p}d_i^{-4}\right)+2\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}d_i^{-2}d_j^{-2}\left(sin^2(\alpha_j-\alpha_i)-1\right)<0 \tag{57}$$

From (55), (56), and (57), we have

$$\sqrt{D^2-4F}=2E-D\geq 0\geq\frac{2F-D^2}{D} \tag{58}$$

Futhermore, we have

$$F-DE\leq 0 \tag{59}$$

Obviously, $GDOP_{\mathrm{AOA}}^{1}$ is an increasing function with respect to r_1, which means $r_{1,\min}$ is the optimal distance of the first CN. Similar conclusion is available for the other CN. After the former m CN have been placed, we have

$$\begin{aligned}\mathfrak{R} &= \sum\nolimits_{j=1}^{m-1}r_{j,\min}^{-2}\cos 2\beta_{\mathrm{O},j,\mathrm{A}}\\ &\quad+\sum\nolimits_{n=1}^{N}d_n^{-2}\cos 2\alpha_n\end{aligned} \tag{60}$$

Then the optimal azimuth of the $(m+1)$th CN to be placed can be determined by

$$\beta_{\mathrm{O},m+1,\mathrm{A}}=\begin{cases}random & if\ \mathfrak{R}=0\\ \beta_{\mathrm{A}}^{1} & if\ \mathfrak{R}<0\\ \beta_{\mathrm{A}}^{2} & if\ \mathfrak{R}>0\end{cases} \tag{61}$$

Particularly, if $r_{1,\min}=r_{2,\min}=\ldots=r_{M,\min}=r_{\min}$, the inertia dependency factor can also be calculated by

$$\begin{cases}\left((Z_{\mathrm{A}}-1)\cdot r_{\min}^{-2}\cos 2\beta_{\mathrm{O},1,\mathrm{A}}+\sum\limits_{n=1}^{N}d_n^{-2}\cos 2\alpha_n\right)\sum\limits_{n=1}^{N}d_n^{-2}\cos 2\alpha_n>0\\ \left(Z_{\mathrm{A}}r_{\min}^{-2}\cos 2\beta_{\mathrm{O},1,\mathrm{A}}+\sum\limits_{n=1}^{N}d_n^{-2}\cos 2\alpha_n\right)\sum\limits_{n=1}^{N}d_n^{-2}\cos 2\alpha_n<0\end{cases} \tag{62}$$

so

$$Z_{\mathrm{A}}=\sqrt{\left(\sum_{n=1}^{N}r_{\min}^{2}d_n^{-2}\sin 2\alpha_n\right)^2+\left(\sum_{n=1}^{N}r_{\min}^{2}d_n^{-2}\cos 2\alpha_n\right)^2} \tag{63}$$

Finally, the solution to the optimal deployment of cooperative nodes in two-dimensional AOA-based localization system can be generalized as follows.

Step 1: According to the spatial relationship between the GN and PT, establish the initial observation function and calculate β_{A}^{1}, β_{A}^{2}, and the initial $\mathfrak{R}$.

Step 2: Choose the lower bound of each r_m as the the optimal distance for each CN.

Step 3: Determine the optimal azimuth of the first CN from β_{A}^{1} and β_{A}^{2}, according to the polarity of $\mathfrak{R}$.

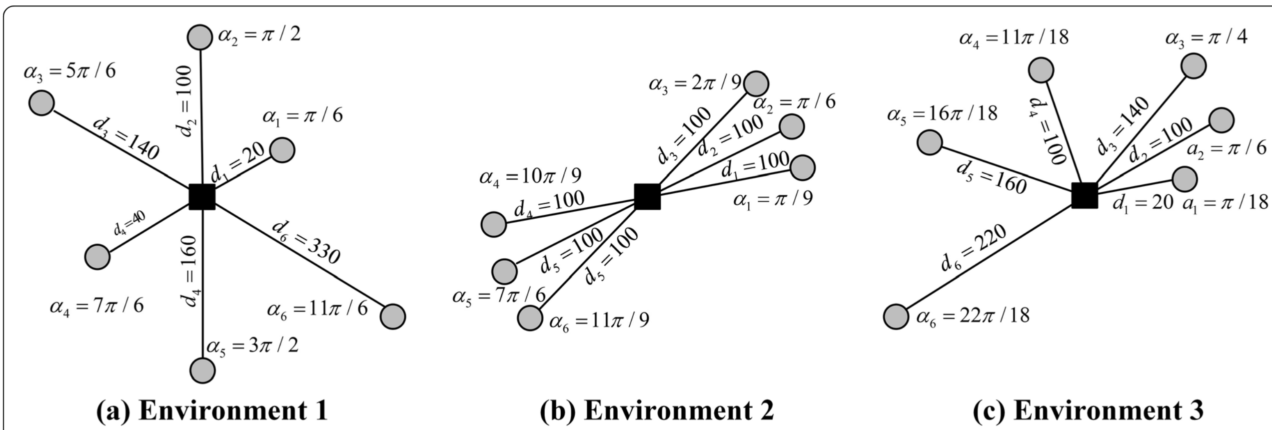

Fig. 3 The layout of the three different simulation environments. **a** Environment 1. **b** Environment 2. **c** Environment 3

Table 1 Optimal performance obtained by OPMPD approach

	Environment 1						Environment 2						Environment 3					
m	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
$r_{m,\min}$	20	20	20	20	20	20	20	20	20	20	20	20	20	15	10	25	18	13
m	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
$r_{m,\max}$	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40

Step 4: If $r_{1,\min} = r_{2,\min} = \ldots = r_{M,\min} = r_{\min}$ is not satisfied, determine the optimal azimuth of the other CN according to the polarity of $\mathfrak{R}$ updated.

Step 5: If $r_{1,\min} = r_{2,\min} = \ldots = r_{M,\min} = r_{\min}$ is satisfied, calculate Z_A. If $Z_A < M$, deploy the whole CNs in the same azimuth as the first CN assigned. If $Z_A \geq M$, deploy the former Z_A CNs in the same azimuth as the first CN assigned while place the (Z_A+1)th CN in the azimuth vertical to that of the first CN. Then, β_A^1 and β_A^2 are alternatively chosen as the optimal azimuth of the rest CNs.

5 Simulation

In this section, we establish three quite different simulation environments to evaluate the performance of OPMPD approach and compare it with partial exhaustive method, global exhaustive method, and eigenvalue-based approach. It indicates that our approach could provide a rapid and accurate deployment of CNs for TOA-based and AOA-based localization system and is superior to the other approaches.

5.1 Simulation set up

These simulations are implemented in MATLAB language and tested on a PC with an Intel Core i5-3470 CPU of 3.20 GHz and DDR3 SDRAM of 8 GB. Suppose there are totally 6 CNs to be placed to improve the accuracy of a PT that serviced by 6 GNs. Simulation data are collected from three environments whose details depicted in Fig. 3 and Table 1. As reported in (31) and (49), if $\mathfrak{R} = 0$, any azimuth can be regarded as the best choice. In our test, for the case of $\mathfrak{R} = 0$, we respectively choose $\pi/5$, $\pi/6$, and $\pi/4$ as the optimal azimuth of CN in environment 1, environment, 2 and environment 3 for easy of analysis. In addition, we set the domain of $\beta_{O,m,T}$ as $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$, considering the periodicity of the $Q(\beta_m)$ and the scope configured in (28).

5.2 Deployment accuracy

Because OPMPD approach estimate the optimal position of each CN based on the partial differentiation and the stepwise strategy, both partial exhaustive method and global exhaustive method are chosen to evaluate the accuracy. The significant difference between them is that the optimum of the CNs in partial exhaustive method is decided step by step just like eigenvalue-based approach and our approach while the optimum of the CNs in global exhaustive method determined all at once. For the two kinds of exhaustive method, the granularity of

Table 2 Optimal performance obtained by OPMPD approach

	Environment 1				Environment 2				Environment 3			
(a) TOA-based												
m	$\mathfrak{R}$	$\beta_{O,m,T}$	$GDOP_{C,TOA}^m$	Z_T	$\mathfrak{R}$	$\beta_{O,m,T}$	$GDOP_{C,TOA}^m$	Z_T	$\mathfrak{R}$	$\beta_{O,m,T}$	$GDOP_{C,TOA}^m$	Z_T
1	0	0.6283	0.7683	1	2.8794	2.9044	1.0308	6	1.6133	2.0051	0.7739	3
2	0.309	2.1991	0.7071		2.3794	2.9044	0.8010		0.9675	2.0051	0.7085	
3	0	0.6283	0.6708		1.8794	2.9044	0.7004		0.3217	2.0051	0.6677	
4	0.309	2.1991	0.6325		1.3794	2.9044	0.6425		−0.3241	0.4343	0.6332	
5	0	0.6283	0.6055		0.8794	2.9044	0.6045		0.3217	2.0051	0.6037	
6	0.309	2.1991	0.5774		0.3794	2.9044	0.5775		−0.3241	0.4343	0.5778	
(b) AOA-based												
m	$\mathfrak{R}$	$\beta_{O,m,A}$	$GDOP_{C,AOA}^m$	Z_A	$\mathfrak{R}$	$\beta_{O,m,A}$	$GDOP_{C,AOA}^m$	Z_A	$\mathfrak{R}$	$\beta_{O,m,A}$	$GDOP_{C,AOA}^m$	Z_A
1	0.0015	2.1057	26.3141	2	0.0002894	2.0944	45.8142	1	0.0024	1.7526	27.4448	
2	0.0002527	2.1057	22.5644		−0.0009206	0.5236	26.8686		0.0002859	1.7526	22.7065	
3	−0.000948	0.5349	19.2462		0.0002894	2.0944	22.8770		−0.0041	0.1868	14.8350	
4	0.0002527	2.1057	17.5197		−0.0009206	0.5236	19.4544		0.0052	1.7526	13.9309	
5	−0.000948	0.5349	15.9077		0.0002894	2.0944	17.6657		0.0037	1.7526	12.8015	
6	0.0002527	2.1057	14.8611		−0.0009206	0.5236	16.0237		0.0008377	1.7526	11.6386	

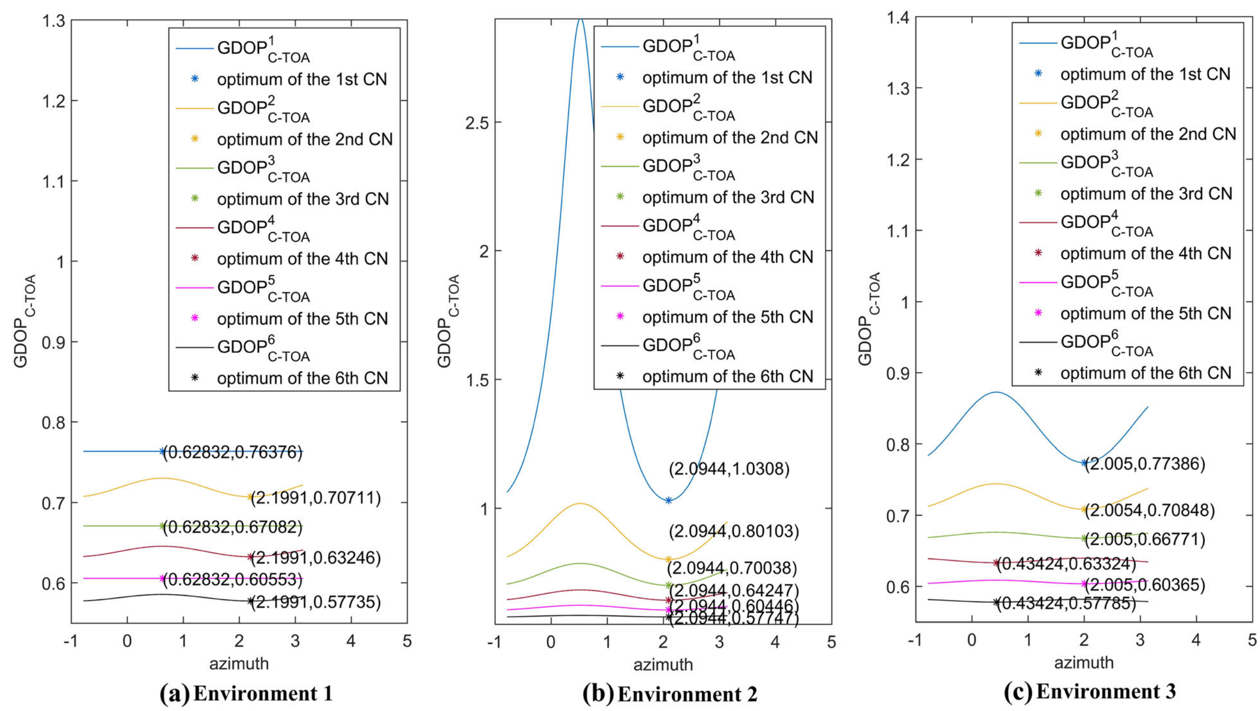

Fig. 4 Performance of partial exhaustive method towards TOA-based system. **a** Environment 1. **b** Environment 2. **c** Environment 3

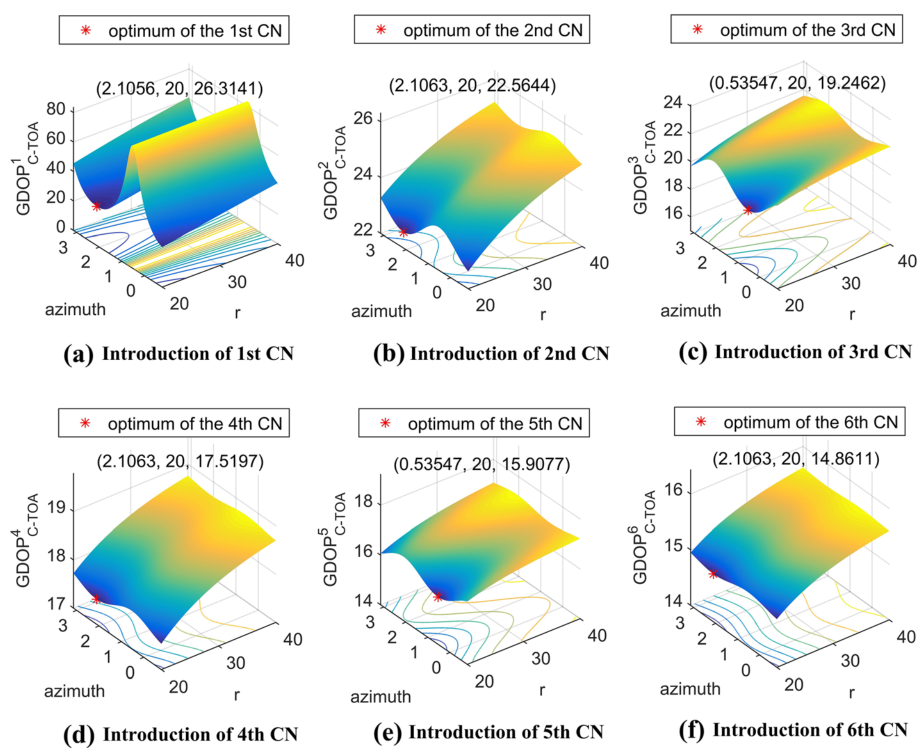

Fig. 5 Performance of partial exhaustive method towards AOA-based system in environment 1. **a** Introduction of the first CN. **b** Introduction of the second CN. **c** Introduction of the third CN. **d** Introduction of the fourth CN. **e** Introduction of fifth CN. **f** Introduction of sixth CN

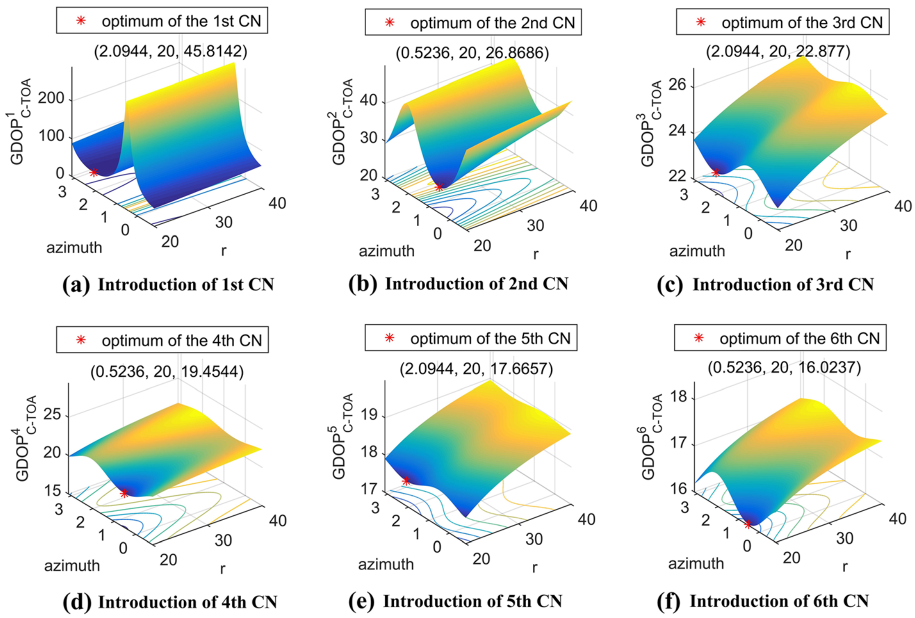

Fig. 6 Performance of partial exhaustive method towards AOA-based system in environment 2. **a** Introduction of first CN. **b** Introduction of second CN. **c** Introduction of third CN. **d** Introduction of fourth CN. **e** Introduction of fifth CN. **f** Introduction of sixth CN

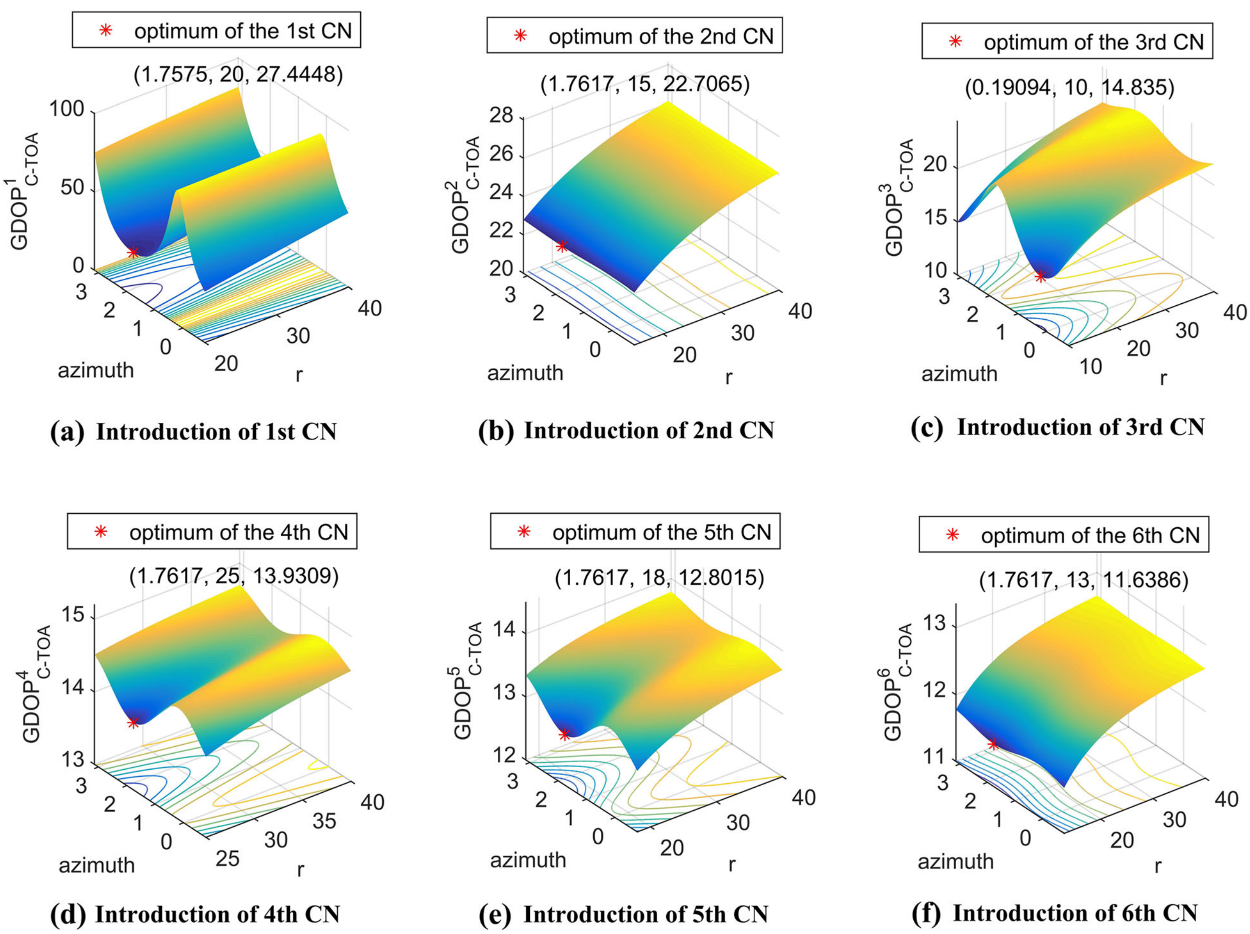

Fig. 7 Performance of partial exhaustive method towards AOA-based system in environment 3. **a** Introduction of first CN. **b** Introduction of second CN. **c** Introduction of third CN. **d** Introduction of fourth CN. **e** Introduction of fifth CN. **f** Introduction of sixth CN

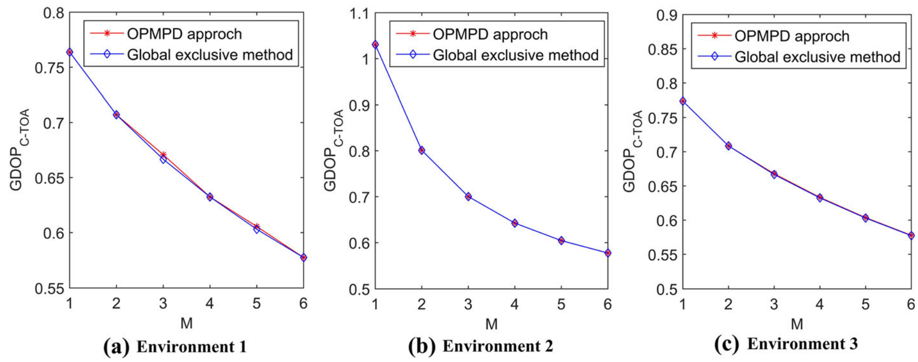

Fig. 8 Accuracy comparison of OPMPD approach and global exclusive method towards TOA-based system. **a** Environment 1. **b** Environment 2. **c** Environment 3

azimuth is set $\pi/3600$ rad and the granularity of distance is set 0.5 m.

Table 2 depicts the optimal performance obtained by OPMPD approach. As revealed in Table 2(a), 0.6283, 2.9044, and 2.0051 are chosen as the optimal azimuth of the first CN in the three environments. Compared to the performance depicted in Fig. 4, the optimum of each CN achieved by OPMPD nearly equals to the result afforded by partial exhaustive method, which proves the correctness of theoretical derivation in OPMPD approach. Similar conclusion about azimuth toward AOA-based localization system can be derived from Table 2(b) and Figs. 5, 6, and 7. Besides, it is showed that the GDOP- of AOA-based cooperative localization system decreases as the distance of the CN shrinks, which agrees with the conclusion in (59).

The accuracy comparison with global exclusive method is illustrated in Figs. 8 and 9. It can be seen that the GDOP of OPMPD approximately equals to that of global exclusive method with occasionally a little larger. Whereas, it is important to note that the OPMPD approach is much less complex.

5.3 Deployment efficiency

Running time is chosen to evaluate the efficiency of the OPMPD approach and the eigenvalue-based approach. For the TOA-based cooperative system, the procedure of the eigenvalue-based approach consists of three steps. First, the symmetric matrix $\mathbf{G}_{\mathrm{TOA}}$ would be calculated. Second, the eigenvalue and eigenvector of $\mathbf{G}_{\mathrm{TOA}}$ should be determined. Third, the optimal position of the CN to be placed is decided between the two eigenvector by contrasting the fore-and-aft eigenvalue. On the other, the procedure in OPMPD approach mainly comprises two steps. First, the positive azimuth factor, the negative azimuth factor, the initial cosine azimuth coefficient, and the inertia dependence factor are calculated. Second, the optimal position of each CN can be obtained according to the polarity of the initial cosine azimuth coefficient and the value of the inertia dependence factor. Figures 10 and 11, respectively, shows the pseudocode for the procedure in eigenvalue-based approach and in OPMPD approach towards TOA-based localization system. The data of running time is collected on the basis of 10,000 times Monte Carlo simulation. As illuminated in Table 3,

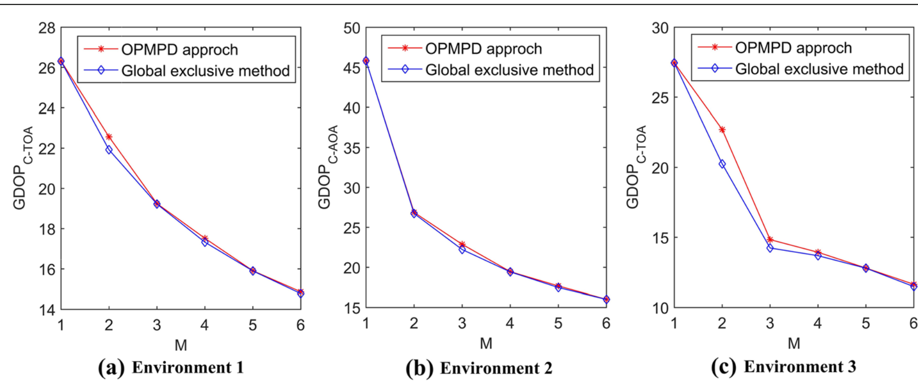

Fig. 9 Accuracy comparison of OPMPD approach and global exclusive method towards AOA-based system. **a** Environment 1. **b** Environment 2. **c** Environment 3

Procedure $\boldsymbol{\alpha}=[\alpha_1,\alpha_2,...,\alpha_n]$;
Initialize optimal azimuth $\beta_{\mathrm{O},m,\mathrm{T}}(m=1,2,3,...M)$ set to 0;
$\boldsymbol{H}_{\mathrm{TOA}}=[-\cos(\boldsymbol{\alpha});-\sin(\boldsymbol{\alpha})]^T$;
$\boldsymbol{G}_{\mathrm{TOA}}=\left(\boldsymbol{H}_{\mathrm{TOA}}{}^{\mathrm{T}}\boldsymbol{H}_{\mathrm{TOA}}\right)^{-1}$;
Acquire eigenvalue λ_1,λ_2 and eigenvector $\boldsymbol{e}_1,\boldsymbol{e}_2$ from $\boldsymbol{G}$ where $\lambda_1<\lambda_2$;
$\beta_{\mathrm{O},1,\mathrm{T}}=\arctan(\boldsymbol{e}_1)$
for i=2,…,M
  if $\lambda_1+1<\lambda_2$
    $\beta_{\mathrm{O},i,\mathrm{T}}=\arctan(\boldsymbol{e}_1)$;
    $\lambda_1=\lambda_1+1$;
  else
    $\beta_{\mathrm{O},i,\mathrm{T}}=\arctan(\boldsymbol{e}_2)$;
    $\lambda_2=\lambda_2+1$;
  end
end
end procedure

Fig. 10 Pseudocode for the deployment procedure in eigenvalue-based approach towards TOA-based localization system

the efficiency of the OPMPD approach is nearly two times higher than that of the eigenvalue-based approach. One reason is that the complex eigen-decomposition of the matrix has been replaced by the straightforward arithmetic operation.

5.4 Requirement of inertia dependence factor

Inertia dependence factor that reveals the inertia and the recursiveness of deployment also accounts for the efficiency improvement. For TOA-based localization system, the requirement of inertia dependence factor is that the CN can be placed in any azimuth. In environment 2, since $Z_{\mathrm{T}}=6$, assigning the total CNs in the azimuth 2.9044 rad would achieve the minimum value of $\mathrm{GDOP}^{6}_{\mathrm{C,TOA}}=0.5775$. For AOA-based localization system, besides azimuth, the requirement of inertia dependence factor includes that the lower bound of the distance to the PT of each CN should be identical to the others'. In environment 1, since $r_{1,\min}=r_{2,\min}=\ldots=r_{6,\min}=20$, $Z_{\mathrm{A}}=2$, thereby assigning the former two

Procedure $\boldsymbol{\alpha}=[\alpha_1,\alpha_2,...,\alpha_n]$;
Initialize optimal azimuth $\beta_{\mathrm{O},m,\mathrm{T}}(m=1,2,3,...M)$ set to 0;
Initialize $\Re=\mathrm{sum}(\cos(2\boldsymbol{\alpha}))$ and $\kappa=\mathrm{sum}(\sin(2\boldsymbol{\alpha}))$ and $Z_{\mathrm{T}}=\left\lceil\sqrt{\Re^2+\kappa^2}\right\rceil$;
$\beta^1_{1,\mathrm{T}}=0.5\times\arctan\left(\frac{\kappa}{\Re}\right)$ and $\beta^2_{1,\mathrm{T}}=\beta^1_{1,\mathrm{T}}+\frac{1}{2}\pi$;
if $\Re<0$
  $\beta_{\mathrm{O},i,\mathrm{T}}(m=1,2,3,...Z_T)=\beta^1_{1,\mathrm{T}}$;
  if $Z_{\mathrm{T}}>M$
    $\beta_{\mathrm{O},j,\mathrm{T}}(j=Z_{\mathrm{T}}+1,Z_{\mathrm{T}}+3,...M)=\beta^2_{1,\mathrm{T}}$;
    $\beta_{\mathrm{O},j,\mathrm{T}}(j=Z_{\mathrm{T}}+2,Z_{\mathrm{T}}+4,...M)=\beta^1_{1,\mathrm{T}}$;
  end
else
  $\beta_{\mathrm{O},i,\mathrm{T}}(m=1,2,3,...Z_{\mathrm{T}})=\beta^2_{1,\mathrm{T}}$;
  if $Z_{\mathrm{T}}>M$
    $\beta_{\mathrm{O},j,\mathrm{T}}(j=Z_{\mathrm{T}}+1,Z_{\mathrm{T}}+3,...M)=\beta^1_{1,\mathrm{T}}$;
    $\beta_{\mathrm{O},j,\mathrm{T}}(j=Z_{\mathrm{T}}+2,Z_{\mathrm{T}}+4,...M)=\beta^2_{1,\mathrm{T}}$;
  end
end
end procedure

Fig. 11 Pseudocode for the deployment procedure in OPMPD approach towards TOA-based localization system

Table 3 Average running time comparison of OPMPD approach and eigenvalue-based approach towards TOA-based localization system

	Environment 1	Environment 2	Environment 3
Eigenvalue-based	32.5028 μs	33.3227 μs	33.5372 μs
OPMPD	11.4669 μs	11.6083 μs	11.6047 μs

CNs in the azimuth 2.1057 rad and alternately choosing 0.5349 rad and 2.1057 rad as the optimal azimuth of each remaining CN would achieve the minimum value of $GDOP_{C,AOA}^{6} = 14.8611$. But in environment 3, as the lower bound of the distance of the CNs differ from each other, inertia dependence factor is unavailable which renders the optimal azimuth of each CN decided by the polarity of the updating cosine azimuth coefficient.

6 Conclusions

In this paper, our research is to solve the optimal deployment of cooperative nodes in two-dimensional localization system. To achieve the deployment with the least GDOP, the GDOP expressions of TOA-based and AOA-based cooperative localization systems are derived on the basis of Cramer-Rao bound. By examining the partial differentiation of the GDOP and leveraging stepwise strategy, an approach termed as OPMPD is suggested to expose the optimal position for each CN. Inertia dependence factor is also deducted which reveals the relationship among the optimal positions of each CN. Simulation results prove that our solution almost achieves the same GDOP to that of global exclusive method but with less time complexity. It is noted that the solution proposed in this paper are only suitable for TOA system and AOA system, not for TDOA system. The reason for this is that the idea of TDOA is to determine the relative position of the mobile transmitter by examining the difference in time at which the signal arrives at multiple measuring units, rather than the absolute arrival time of TOA. The Jacobian matrix of TDOA is quite different from those of TOA and AOA. Hence, our future work is to study the optimal deployment of cooperative nodes in other more complex localization systems such as the TDOA-based, the TDOA/TOA-based, and the TDOA/AOA-based.

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Authors' contributions
WS and XQ conceived and designed the study. JL, SY and LC performed the simulation experiments. YY and XF wrote the paper. WS and XQ reviewed and edited the manuscript. All authors read and approved the manuscript.

Competing interests
The authors declare that they have no competing interests.

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References
1. A Fehske, G Fettweis, J Malmodin et al., The global footprint of mobile communications: the ecological and economic perspective. Communications Magazine, IEEE **49**(8), 55–62 (2011)
2. AH Sayed, A Tarighat, N Khajehnouri, Network-based wireless location: challenges faced in developing techniques for accurate wireless location information. Signal Processing Magazine, IEEE **22**(4), 24–40 (2005)
3. D Liu, B Sheng, F Hou et al., From wireless positioning to mobile positioning: an overview of recent advances. Systems Journal, IEEE **8**(4), 1249–1259 (2014)
4. PK Enge, The global positioning system: signals, measurements, and performance. Int. J. Wireless Inf. Networks **1**(2), 83–105 (1994). Modeling and analysis for the GPS pseudo-range observable,1995
5. X Zhiyang, S Pengfei, Wireless location determination for mobile objects based on GSM in intelligent transportation systems. Systems Engineering and Electronics, Journal of **14**(2), 8–13 (2003)
6. D Peral-Rosado, J Lopez-Salcedo, G Seco-Granados et al., *Achievable localization accuracy of the positioning reference signal of 3GPP LTE[C]//Localization and GNSS (ICL-GNSS), 2012 International Conference on* (IEEE, 2012), pp. 1–6
7. A Bours, E Cetin, AG Dempster, Enhanced GPS interference detection and localisation. Electron. Lett. **50**(19), 1391–1393 (2014)
8. DKP Tan, H Sun, Y Lu et al., Passive radar using global system for mobile communication signal: theory, implementation and measurements. Radar, Sonar and Navigation, IEE Proceedings. IET **152**(3), 116–123 (2005)
9. S Wing, Mobile and wireless communication: space weather threats, forecasts, and risk management. IT Professional **5**, 40–46 (2012)
10. YL Yeh, KC Cheng, WH Wang et al., Very short-term earthquake precursors from GPS signal interference based on the 2013 Nantou and Rueisuei earthquakes, Taiwan[J]. J Asian Earth Sci, **114**,312-320(2015)
11. MH MacAlester, W Murtagh, Extreme space weather impact: an emergency management perspective. Space Weather **12**(8), 530–537 (2014)
12. J Meguro, T Murata, J Takiguchi et al., GPS multipath mitigation for urban area using omnidirectional infrared camera. Intelligent Transportation Systems, IEEE Transactions on **10**(1), 22–30 (2009)
13. Y Qi, H Kobayashi, H Suda, On time-of-arrival positioning in a multipath environment. Vehicular Technology, IEEE Transactions on **55**(5), 1516–1526 (2006)
14. I Suberviola, I Mayordomo, J Mendizabal, Experimental results of air target detection with a GPS forward-scattering radar. Geoscience and Remote Sensing Letters, IEEE **9**(1), 47–51 (2012)
15. AJ Fehske, F Richter, GP Fettweis, *Energy efficiency improvements through micro sites in cellular mobile radio networks[C]//GLOBECOM Workshops, 2009 IEEE* (IEEE, 2009), pp. 1–5
16. S McAleavey, Ultrasonic backscatter imaging by shear-wave-induced echo phase encoding of target locations. Ultrasonics, Ferroelectrics, and Frequency Control, IEEE Transactions on **58**(1), 102–111 (2011)
17. WY Chiu, BS Chen, CY Yang, Robust relative location estimation in wireless sensor networks with inexact position problems. Mobile Computing, IEEE Transactions on **11**(6), 935–946 (2012)
18. R Faragher, R Harle, Location fingerprinting with bluetooth low energy beacons. IEEE Journal on Selected Areas in Communications **33**(11), 2418–2428 (2015)
19. L Yang, Y Chen, XY Li et al., *Tagoram: Real-time tracking of mobile RFID tags to high precision using COTS devices[C]//Proceedings of the 20th annual international conference on Mobile computing and networking* (ACM, 2014), pp. 237–248
20. K Yu, J Montillet, A Rabbachin et al., UWB location and tracking for wireless embedded networks. Signal Processing **86**(9), 2153–2171 (2006). 超宽带
21. Z He, Y Ma, R Tafazolli, *Accuracy limits and mobile terminal selection scheme for cooperative localization in cellular networks[C]//Vehicular Technology Conference (VTC Spring), 2011 IEEE 73rd* (IEEE, , 2011), pp. 1–5
22. RM Vaghefi, RM Buehrer, Cooperative localization in NLOS environments using semidefinite programming. Communications Letters, IEEE **19**(8), 1382–1385 (2015)

23. W Li, Y Hu, X Fu et al., Cooperative positioning and tracking in disruption tolerant networks. Parallel and Distributed Systems, IEEE Transactions on **26**(2), 382–391 (2015)
24. H Liu, H Darabi, P Banerjee et al., Survey of wireless indoor positioning techniques and systems. Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on **37**(6), 1067–1080 (2007)
25. FL Piccolo, *A new cooperative localization method for UMTS cellular networks[C]//Global Telecommunications Conference, 2008. IEEE GLOBECOM 2008. IEEE. IEEE*, 2008, pp. 1–5
26. DJ Torrieri, Statistical theory of passive location systems. IEEE Transactions on Aerospace and Electronic Systems **20**(2), 183–198 (1984)
27. N Levanon, Lowest GDOP in 2-D scenarios. IEE Proceedings-radar, sonar and navigation **147**(3), 149–155 (2000)
28. AG Dempster, Dilution of precision in angle-of-arrival positioning systems. Electronics Letters **42**(5), 291–292 (2006)
29. P Deng, J Yu Li, GDOP performance analysis of cellular location system. Journal of southwest jiaotong university **40**(2), 184–188 (2005)
30. Q Quan, Low bounds of the GDOP in absolute-range based 2-D wireless location systems. Information Science and Digital Content Technology (ICIDT), 2012 8th International Conference on. IEEE **1**, 135–138 (2012)
31. XW Lv, KH Liu, P Hu, Efficient solution of additional base stations in time-of-arrival positioning systems. Electronics letters **46**(12), 861–863 (2010)