RESEARCH Open Access

# CrossMark

# When is it optimal for the event-triggered strategy in energy harvesting transmission

Yinxi Tan , Shanyun Liu and Pingyi Fan\*

#### **Abstract**

The energy harvesting network in mobile environment is promising for sustainable environment and green communication. How to efficiently use the harvested energy is a key to improve the performance of energy harvesting communication network. This paper considers a point-to-point communication scenario where a static sensor harvests energy and transmits information to a moving agent. From the viewpoint of information theory, we present an optimal power allocation (PA) solution to achieve the maximum channel service. We prove that using a water-filling transmission power, during a single transmission timeslot, the energy depletion time is uniquely determined by the initial energy in the sensor regardless of the energy-harvesting power. On the other hand, the energy depletion time is a determinant of whether the energy is exhausted at the end of the transmission timeslot. Based on the obtained results, an event-triggered transmission strategy is proposed where a sensor harvests energy to a certain amount so as to trigger information transmission towards the moving agent. Some numerical results are provided to confirm the theoretical analysis and our proposed event-triggered implementation approach.

**Keywords:** Energy harvest, Mobile communication, Water filling, Event-triggered strategy, Channel service

#### 1 Introduction

In future networks, it may become more popular for a remote node using renewable energy to transfer information. The energy harvesting node not only saves non-renewable energy from depletion but also supplies power for many automatic unattended operation circumstances. Different from conventional batteries, energy harvesting enables information source to replenish energy automatically. However, due to the complication of the channel and the noise, the transmission efficiency is the principal concern for putting energy harvesting in wide application for mobile communication. In order to meet these requirements, it is desiderative to study the optimal strategy for energy harvesting network in mobile communication systems.

There are a multitude of literatures working on energy harvesting in several aspects [1]. In [2], minimizing transmission completion time is considered given a number of bits to send where packets are ready before transmission or arrives during transmission. Reference [3, 4] maximize throughput where the framework of [3] invents

the shortest path within a narrowing tunnel as optimal policy and [4] discusses when energy sources are time varying in point-to-point wireless communication with channel fading. Similar to maximizing throughput, in [5–8], simultaneous wireless information and power transfer is studied to maximize the achievable information rate. Reference [9, 10] consider both minimizing transmission completion time and maximizing throughput. In [9], it is demonstrated that the problem of maximizing throughput can be transformed to minimizing transmission time equivalently. Reference [10] further explores the stochastic fading channel and adapts directional waterfilling and stochastic dynamic programming to solve the on-line optimization.

Besides transmission time and throughput, there are other metrics that can measure the performance of an energy-harvesting transmission system. Reference [11] considers the group cooperation in the wireless powered communication system in order to maximize the weighted sum-rate and to minimize the total consumed power. In [12], a balance policy is proposed for adapting transmission probability to harvesting rate when energy supply is

<sup>\*</sup>Correspondence: fpy@mail.tsinghua.edu.cn Department of Electronic Engineering, Tsinghua University, Beijing, China



time correlated. Reference [13] maximizes the long-term average transmitted data importance applying threshold policy. In [14], the mean delay optimal energy management policies are identified considering fading channel and the leakage of energy, where greedy policy puts up a good performance in low SNR regime. Also, in [15], a general reward is derived to evaluate the performance of different design objectives where energy supply is memoryless. In [16], minimization of the energy requirement is considered to achieve green system design.

All the works aforementioned except [3] study the energy-harvesting problems where energy packets are in discrete forms. From the view of practical applications, [3, 17–21] consider the moving agent when energy harvesting happens. When the energy harvested from environment is consecutive, channel service becomes a more suitable performance metric as [19] considered. Literature [20] gives feasibility to such continuous scene. It derives constant power allocation schemes in terms of channel service maximization. In [21], the trade-off between the channel service and the system fairness is discussed.

This paper mainly focuses on answering the problem when it is optimal for the event-triggered strategy in energy-harvesting transmission. As an extension of our previous work in [22], we consider an energy-harvesting sensor system from which a mobile agent received information. For example, there are some specific scenarios like a bus or a car gathers information from sensor network while driving along the highway. We employ the channel service as the metric where the sensors harvest energy and transmit information at the same time. We model the system as an infinite battery capacity model, and the sensor immediately starts to transmit information when the mobile information collection agent, also referred to mobile agent, is coming. Under these assumptions, we calculate the precise maximum channel service.

Different from the literatures above, we introduce the event-triggered scheduling [23] in our strategy, in order to approach the maximum channel service. In another word, we fix attention on a channel service maximization problem with infinite battery in a mobile scenario where energy harvesting is simultaneous with information transmission.

The remainder of this paper is arranged as follows: Section 2 introduces a one-dimensional mobility system model and formulates the problem. In Section 3, we give a general implicit form of the optimal solution in subsection 3.1 and discuss the solution in detail in subsection 3.2. Subsection 3.3 derives event-triggered transmission strategy and the corresponding algorithm based on water filling. Section 4 demonstrates the performance of our strategy and discusses the numerical results. Section 5 concludes the paper finally.

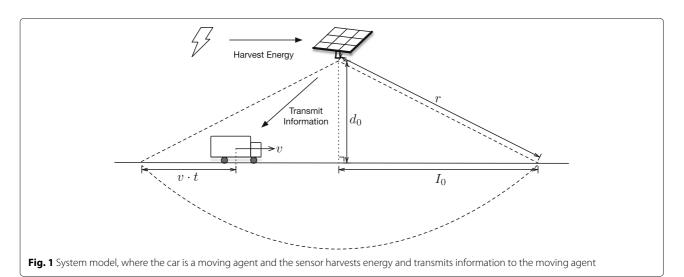
#### 2 System model

Consider the scenario that a static sensor node harvests energy and transmits information to a moving node, as shown in Fig. 1. Due to signal attenuation, we assume that transmission can only happen when the distance between the two nodes is smaller than or equal to a threshold, r. As the moving node passes by along a straight line, the distance between the moving node and the sensor node d(t) is

$$d(t) = \sqrt{(L_0 - vt)^2 + d_0^2}. (1)$$

where  $\nu$  is the velocity of the moving node, t is the instantaneous time,  $L_0$  is the maximum transmission range on one side, and  $d_0$  is the minimum distance between the agent and the static sensor. Figure 1 also shows the threshold r, also called effective range, that satisfies

$$r^2 = L_0^2 + d_0^2. (2)$$



This effective range r contains all the positions of the moving node where the transmission can be conducted. Because of the symmetry of the trajectory, the wireless channel varies symmetrically in time, with AWGN noise. We are going to study the power allocation (PA) strategy in this transmission procedure.

Let W denote the bandwidth,  $N_c(t)$  denote the noise power,  $G_s$  denote the channel gain, and  $P_s(t)$  denote the transmission power which is determined by a certain PA strategy. The SNR is

SNR = 
$$\frac{G_s P_s(t)}{N_c(t)} = \frac{P_s(t)}{N_c(t)/G_s} = \frac{P_s(t)}{N(t)},$$
 (3)

where  $N(t) = N_c(t)/G_s$  is denoted as the equivalent noise power in this paper for convenience. Let  $N_0$  represent the noise power density, and  $\alpha_c$  represent the attenuation coefficient of the channel, the instantaneous channel capacity C(t) [24] is given by

$$C(t) = W \log_2 \left( 1 + \frac{P_s(t)}{N(t)} \right), \tag{4}$$

where N(t) is the equivalent noise depended on the distance, given as

$$N(t) = WN_0 d^{\alpha_c}(t)/G_s. \tag{5}$$

Moreover, the channel service in a transmission procedure is defined as

$$S = \int_{T_s} C(\tau) d\tau, \tag{6}$$

where  $T_s$  is the time interval of the transmission. In our case,  $T_s = \left[0, \frac{2L_0}{\nu}\right]$ . It is known that the optimal strategy which achieves maximum channel service without energy harvesting is the water-filling strategy [21].

However, in the energy harvesting scenario, we have a new energy causality constraint. Let B(t) denote the total energy that the sensor node can hold at time t, without supplying power for information transmission. B(t) is composed of two parts, the initial energy in the battery  $B_0$  and the harvest energy in the process of transmission. Assuming that the power of energy harvesting is a con-

stant during the process of transmission, denoted as  $P_h$ , and the battery capacity is sufficient large, we have

$$B(t) = B_0 + P_h t. (7)$$

On the other hand, define  $E_s(t)$  as the total energy that is consumed during the transmission. We have

$$E_s(t) = \int_0^t P_s(\tau) d\tau. \tag{8}$$

Then, the energy causality constraint is

$$E_s(t) \le B(t), \ \forall t \in T_s.$$
 (9)

This constraint ensures that energy which is harvested in the future can not be consumed now. Inequality (9) must be fulfilled by any feasible power allocation solution.

So far, the optimization problem to maximize channel service in a transmission procedure is given by

$$\max_{P_s(t)} \int_0^{\frac{2L_0}{\nu}} W \log_2 \left( 1 + \frac{P_s(t)}{N(t)} \right) dt,$$
s.t. 
$$\int_0^t P_s(\tau) d\tau \le B(t), \forall t \in \left[ 0, \frac{2L_0}{\nu} \right],$$

$$P_s(t) \ge 0, \forall t \in \left[ 0, \frac{2L_0}{\nu} \right].$$
(10)

where

$$N(t) = WN_0 d^{\alpha_c}(t)/G_s$$

and

$$d(t) = \sqrt{(L_0 - vt)^2 + d_0^2}.$$

In the next section, we will characterize the solution of the optimization problem and corresponding transmission strategies.

# 3 Power allocation in the energy-harvesting scenario

In this section, we characterize the solution in the energyharvesting scenario. Based on the properties of the optimal solution, we put forward a corresponding transmission strategy.

## 3.1 The optimal solution: an implicit expression

To obtain the optimal solution to problem (10), we use the Lagrange multiplier method. The Lagrangian function is

$$F = \int_0^{\frac{2L_0}{\nu}} W \log_2 \left( 1 + \frac{P_s(\tau)}{N(\tau)} \right) d\tau - \lambda(t) \left[ \int_0^t P_s(\tau) d\tau - P_h t - B_0 \right], \tag{11}$$

where  $\lambda(t)$  represents the Lagrange multiplier at time t. Note that  $\lambda(t)$  is a function of time variable t rather than a

constant, which differs significantly from discrete works. Set the derivative of F with respect to  $P_s(\tau)$  as zero

$$\frac{\partial F}{\partial P_s(\tau)} = \frac{W}{\ln 2} \frac{P_s(\tau)}{N(\tau) + P_s(\tau)} - \int_{\tau}^{\frac{2L_0}{\nu}} \lambda(t) dt = 0.$$
(12)

Define  $\Lambda(\tau) \triangleq \int_{\tau}^{\frac{2L_0}{\nu}} \lambda(t) dt$ . If  $\Lambda(\tau) \neq 0$ , the solution is given by

$$P_s(\tau) = \left[ \frac{W}{\ln 2 \int_{\tau}^{\frac{2L_0}{\nu}} \lambda(t) dt} - N(\tau) \right]^+. \tag{13}$$

Based on the KKT conditions, we can obtain

$$\lambda(t) \left[ \int_0^t P_s(\tau) \, d\tau - P_h t - B_0 \right] = 0 \tag{14}$$

Note that the multiplier  $\lambda(t)$  is a function of t, (13) associated with the constraint in (14) provides an implicit optimal solution for the channel service maximization problem (10). To get more insights from the solution (13), in section 3.2, we embark from the relaxation of the causality constraint and figure out a feasible optimal solution. Then in section 3.3, we present a corresponding transmission strategy from the angle of event-triggered scheduling.

## 3.2 Characterize the optimal solution

Now, we relax the energy causality constraint. Instead of constraint (9), we only confine the total energy consumed in a transmission procedure, i.e.

$$\int_{0}^{\frac{2L_{0}}{\nu}} P_{s}(\tau) d\tau \leq E_{m},\tag{15}$$

where  $E_m$  is the final state of the energy Eq. (7), given by

$$E_m = B_0 + P_h \frac{2L_0}{\nu}. (16)$$

So the optimization problem (10) becomes

$$\max_{P_s(t)} \int_0^{\frac{2L_0}{\nu}} W \log_2 \left( 1 + \frac{P_s(t)}{N(t)} \right) dt,$$

$$s.t. \int_0^{\frac{2L_0}{\nu}} P_s(\tau) d\tau \le B_0 + P_h \frac{2L_0}{\nu},$$

$$P_s(t) \ge 0, \ \forall t \in \left[ 0, \frac{2L_0}{\nu} \right].$$
(17)

Apparently, problem (17) is a relaxation of problem (10). In this case, the optimal strategy is the water-filling power allocation [21] and the transmission power is given by

$$P_s(t) = \left[\frac{W}{\lambda} - N(t)\right]^+,\tag{18}$$

where  $[\cdot]^+ = \max\{0, \cdot\}.$ 

This is a classic power allocation strategy for maximizing channel service named from a vivid description of the procedure. The power of equivalent noise N(t) is a bowllike curve, and the water-filling strategy shows that the power of the receiving information  $N(t) + P_s(t)$  should be a horizontal line, just like filling water into the bowl.

However, the water-filling solution above can not apply to the real-time energy-harvesting scene, problem (10), because of the absence of causality constraint. In fact, the sensor cannot use the energy which has not harvested yet.

Back to the original energy constraint  $B(t) = P_h t + B_0$ , now we consider the feasibility and optimality of the "water-filling-like" strategies, i.e., set  $P_s(t) = \left[\frac{W}{\lambda} - N(t)\right]^+$  for some  $\lambda > 0$ .

**Theorem 1** Setting  $P_s(t) = \left[\frac{W}{\lambda} - N(t)\right]^+$  for some  $\lambda > 0$  on  $t \in \left[0, \frac{2L_0}{\nu}\right]$ , one can achieve the maximum channel service if and only if  $B(t) = P_h t + B_0$  is the tangent line of  $E_s(t) = \int_0^t P_s(\tau) d\tau$ .

*Proof* See Appendix A. 
$$\Box$$

The examples of Theorem 1 are shown in Fig. 8. A special case of power allocation seen from the figure is that the energy constraint is a constant and the tangent point is the time when transmission ends.

It is easy to observe that the tangent point is the time when  $B(t) = E_s(t)$ . In other words, the battery is depleted at the tangent point. Since the transmission power can maintain zero for some time both at the beginning and the end of the timeslot, in order to simplify the issue, we shall consider the situation where the transmission power is non-zero during the whole timeslot.

**Lemma 1** Consider problem (10). For a water-filling-like strategy, the depletion time when the energy in the battery is zero, denoted by  $t_0$ , is uniquely decided by the initial battery capacity  $B_0$  and irrelevant to the energy-harvesting power  $P_h$ , if and only if

$$\forall t \in \left(0, \frac{2L_0}{\nu}\right), P_s(t) > 0 \tag{19}$$

where 
$$P_s(t) = \left[\frac{W}{\lambda} - N(t)\right]^+$$
.

**Lemma 2** For a water-filling-like strategy, the time when the energy in the battery is depleted, denoted as  $t_0$ , must satisfy  $t_0 > \frac{L_0}{\nu}$ .

*Proof* Since  $B_0 = N(t_0)t_0 - \int_0^{t_0} N(t)dt$  and N(t) is monotone decreasing on interval  $\left[0, \frac{L_0}{\nu}\right]$ , it is seen that  $N(t_0)t_0 \leq \int_0^{t_0} N(t)dt$  when  $t_0 \in \left[0, \frac{L_0}{\nu}\right]$ . Therefore, if a sensor exhausts its energy during the first half of the transmission, we will get  $B_0 \leq 0$ , which yields a contradiction.

According to Lemma 1, although we can not get a closed form for the energy depletion time  $t_0$ , we can still characterize its relation with  $B_0$ .

**Lemma 3** For a water-filling-like strategy, the energy depletion time  $t_0$  is monotonically increasing with the initial energy  $B_0$ .

*Proof* Lemma 2 shows  $t_0 \in \left(\frac{L_0}{\nu}, \frac{2L_0}{\nu}\right]$ . Take the partial derivative of  $B_0$ 

$$\frac{\partial B_0}{\partial t_0} = \frac{\left[\frac{\partial N(t_0)t_0 - \int_0^{t_0} N(t)dt}{\partial t_0}\right]}{\partial t_0}$$

$$= WN_0 t_0 \frac{\partial d^{\alpha_c}(t_0)}{\partial t_0} > 0, \ t_0 \in \left(\frac{L_0}{\nu}, \frac{2L_0}{\nu}\right], \quad (20)$$

which concludes that  $t_0$  is increasing with the initial energy  $B_0$ .

Later, we will prove for a water-filling-like strategy to be optimal, the energy should be depleted at the end of the transmission, meaning Eq. (42). Then, the corresponding initial energy is

$$B_{0} = N\left(\frac{2L_{0}}{\nu}\right) \frac{2L_{0}}{\nu} - \int_{0}^{\frac{2L_{0}}{\nu}} N(t)dt,$$

$$= \int_{0}^{\frac{2L_{0}}{\nu}} \left[ N\left(\frac{2L_{0}}{\nu}\right) - N(t) \right] dt.$$
(21)

Define  $P_{\text{fill}}(t) \triangleq N\left(\frac{2L_0}{\nu}\right) - N(t)$ . According to Lemmas 1, 2, and 3, we have the following theorem

**Theorem 2** The policy  $P_s(t) = \left[\frac{W}{\lambda} - N(t)\right]^+$  with a proper  $\lambda > 0$  can be optimal for problem (10) if and only if

$$B_0 \ge \int_0^{\frac{2L_0}{\nu}} P_{fill}(t)dt,\tag{22}$$

with  $P_{fill}(t) = N(0) - N(t)$  defined on interval  $t \in \left[0, \frac{2L_0}{v}\right]$ .

*Proof* See Appendix C. 
$$\Box$$

So far, we have found a condition under which the maximal channel service can be achieved by a simple water-filling strategy. The condition is

$$B_0 \ge \int_0^{\frac{2L_0}{\nu}} P_{\text{fill}}(t) dt$$

In order to meet this requirement, before the transmission starts, the sensor should harvest at least  $\int_0^{2L_0} P_{\rm fill}(t) dt$  energy. Under such conditions, the sensor can utilize all the energy it harvests before and during the transmission procedure to communicate with the moving agent and achieve the maximal channel service finally.

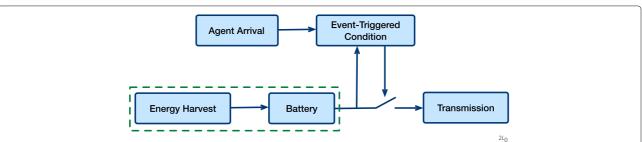
## 3.3 The event-triggered transmission strategy

Section 3.2 shows that if the initial energy in the battery of the sensor node satisfies (22), then a water-filling strategy can achieve the maximal channel service. This enlightens us on an event-triggered strategy as shown in Fig. 2. When the mobile agent arrives, the transmission is triggered if and only if inequality (22) is satisfied.

Before the transmission is triggered, the sensor only harvests energy and does not provide service for transmit-

ting information. Let  $T_{hmin} = \frac{\int_0^{2L_0} P_{\rm fill}(t)dt}{P_h}$ , where  $T_{hmin}$  represents the minimum value of charging time before the transmission is triggered. Thus, the energy-harvesting time  $T_h$  before the transmission is triggered satisfies

$$T_h = \frac{B_0}{P_h} \ge T_{hmin} = \frac{\int_0^{\frac{2L_0}{\nu}} [N(0) - N(t)] dt}{P_h}.$$
 (23)



**Fig. 2** Event-triggered strategy framework. The transmission will be triggered when the battery is charged at least  $\int_0^{\frac{20}{v}} P_{\text{fill}}(t)dt$  as well as the moving agent arrives at the boundary of the effective range

Meanwhile, the transmission time  $T_s$  lasts for  $\frac{2L_0}{\nu}$ , so the service time T is the sum of the energy-harvesting time and the transmission time

$$T = T_h + T_s \ge \frac{\int_0^{\frac{2L_0}{\nu}} [N(0) - N(t)] dt}{P_h} + \frac{2L_0}{\nu}.$$
 (24)

In a service session, the sensor node begins with an empty battery and remains silent for  $T_{hmin}$ . Then, the sensor starts to detect moving agents while it continues to harvest energy. When a moving agent comes, if the trans-

mission is triggered, i.e.,  $B(t) \ge \int_0^{2L_0} P_{\text{fill}}(t) dt$ , the energy harvested in this service session will be exploited. After that, a new session will start. Meanwhile, a transmission is triggered if the energy in the battery when a moving agent arrives satisfies (22). Algorithm 1 shows the details of the event-triggered transmission strategy.

# Algorithm 1 Event-Triggered Transmission Strategy

**Input:** Energy harvesting power  $P_h$ . Environment factors  $L_0$ ,  $d_0$ ,  $G_s$ , W,  $N_0$ ,  $\alpha_c$ .

The maximum velocity of the moving node  $v_{max}$ obtained from the traffic condition.

**Output:** The transmission power  $P_s(t)$ .

**Initialize:** Set  $P_s(t) = 0$ .

1: Set the equivalent noise

$$N(t; \nu) = \frac{WN_0}{G_s} \left[ (L_0 - \nu t)^2 + d_0^2 \right]^{\frac{\pi}{2}}$$

 $N(t;\nu)=rac{WN_0}{G_s}igl[(L_0-\nu t)^2+d_0^2igr]^{rac{a_c}{2}}.$  2: For  $\nu=
u_{
m max}$ , set the minimum equivalent noise as  $\hat{N}(t) = N(t; \nu_{\text{max}}).$ 

3: **loop** 

4: Harvest energy until 
$$B_0 = \int_0^{\frac{2L_0}{\nu_{\text{max}}}} \left[ \hat{N}(0) - \hat{N}(t) \right] dt$$
.

Detect the arrival of the agent and get its velocity  $\nu$ .

Continue harvesting energy  $B(t) = B_0 + P_h \Delta t$ . 6:

7:

Set  $P_{\text{fill}}(t) = \hat{N}(0) - N(t; \nu)$ . if  $B(t) \ge \int_0^{2L_0} P_{\text{fill}}(t) dt$  then  $\triangleright$  Trigger transmission 8:

Set  $E_s = P_h \left( t + \frac{2L_0}{\nu} \right)$ .

Set  $P_s(t) = \frac{\dot{W}}{\lambda} - N(t)$  where  $\lambda$  is given by 10:  $\int_0^{\frac{2L_0}{\nu}} P_s(t)dt = E_s.$ 

11:

9:

12: continue

end if 13:

14: end loop

From Algorithm 1, the utilization of a sensor node is given by

$$\eta = \frac{T_s}{T} \le \frac{2L_0 P_h}{\nu \int_0^{\frac{2L_0}{\nu}} [N(0) - N(t)] dt + 2L_0 P_h}.$$
 (25)

In this case, moving agents may be missed if the transmission is not triggered. This can be easily solved by adding sensor nodes, based on the arrival rate of the moving agents.

Furthermore, we show how the velocity of the agent and the effective range affect the triggering of transmissions. Define the initial energy boundary as  $B_0^*$ 

$$B_0^* \triangleq \int_0^{\frac{2L_0}{\nu}} P_{\text{fill}}(t)dt = \int_0^{\frac{2L_0}{\nu}} [N(0) - N(t)] dt.$$

Take the partial derivative of  $B_0^*$  with respect to  $L_0$ 

$$\frac{\partial B_{0}^{*}}{\partial L_{0}} = \frac{\partial \left[\frac{2WN_{0}L_{0}}{\nu} \left(d_{0}^{2} + L_{0}^{2}\right)^{\frac{\alpha_{c}}{2}}\right]}{\partial L_{0}} \\
- \frac{\partial \left[WN_{0} \int_{0}^{\frac{2L_{0}}{\nu}} \left(d_{0}^{2} + (L_{0} - \nu t)^{2}\right)^{\frac{\alpha_{c}}{2}} dt\right]}{\partial L_{0}} \\
= \frac{2WN_{0}}{\nu} \frac{\partial \left[L_{0} \left(d_{0}^{2} + L_{0}^{2}\right)^{\frac{\alpha_{c}}{2}}\right]}{\partial L_{0}} \\
- WN_{0} \frac{\partial \left[\frac{1}{\nu} \int_{-L_{0}}^{L_{0}} \left(d_{0}^{2} + u^{2}\right)^{\frac{\alpha_{c}}{2}} du\right]}{\partial L_{0}} \\
= \frac{2WN_{0}L_{0}^{2}\alpha_{c}}{\nu} \left(d_{0}^{2} + L_{0}^{2}\right)^{\left(\frac{\alpha_{c}}{2} - 1\right)} > 0,$$
(26)

which indicates that the initial energy boundary monotonically decreases with the effective range. On the other hand, in order to take the partial derivative of  $B_0^*$  with respect to the agent's velocity  $\nu$ , we rewrite  $B_0^*$  as

$$B_{0}^{*} = \frac{2L_{0}WN_{0}}{\nu} \left(d_{0}^{2} + L_{0}^{2}\right)^{\frac{\alpha_{c}}{2}} - WN_{0} \int_{0}^{\frac{2L_{0}}{\nu}} \left[d_{0}^{2} + (L_{0} - \nu t)^{2}\right]^{\frac{\alpha_{c}}{2}} dt$$

$$= \frac{2L_{0}WN_{0}}{\nu} \left(d_{0}^{2} + L_{0}^{2}\right)^{\frac{\alpha_{c}}{2}} - WN_{0} \int_{0}^{2L_{0}} \frac{1}{\nu} \left[d_{0}^{2} + (L_{0} - u)^{2}\right]^{\frac{\alpha_{c}}{2}} du$$

$$= \left[\int_{0}^{2L_{0}} \left(\left(d_{0}^{2} + L_{0}^{2}\right)^{\frac{\alpha_{c}}{2}} - \left(d_{0}^{2} + (L_{0} - u)^{2}\right)^{\frac{\alpha_{c}}{2}}\right) du\right]$$

$$\cdot \frac{WN_{0}}{\nu}.$$
(27)

Set 
$$K = \int_0^{2L_0} \left[ \left( d_0^2 + L_0^2 \right)^{\frac{\alpha_c}{2}} - \left( d_0^2 + (L_0 - u)^2 \right)^{\frac{\alpha_c}{2}} \right] du$$
.  
Since  $\forall u \in [0, 2L_0]$ ,  $K > 0$ , we have

$$\frac{\partial B_0^*}{\partial \nu} = -\frac{KWN_0}{\nu^2} < 0,\tag{28}$$

which shows that the initial energy boundary monotonically decreases with the velocity of the agent.

In conclusion, (26) and (28) show that the initial energy boundary for triggering the transmission increases with the effective range and decreases with the velocity of the moving agent. Also, there is a trade-off between the triggered boundary  $B_0^*$  and the channel service S. Higher boundary brings greater channel service but in turn increases the blocking probability.

#### 4 Numerical results

In this section, we simulate the event-triggered transmission strategy proposed in Section 3 and show numerical results. We set the system power gain as  $G_s = 20$  dB, the attenuation coefficient as  $\alpha_c = 3$ , and the noise power density as  $N_0 = 0.1$ . For convenience, we standardize the system bandwidth as W = 1. Assuming that the initial distance between the static sensor node and the moving track is  $d_0 = 100 \, m$ , the effective interval is  $L_0 = 625 \, m$ , and the velocity of the moving node is  $v = 20 \, m/s$ ; thus, the time interval of the transmission is  $T_s = 62.5 \, \text{s}$ .

In Fig. 3, we plot the energy consulting curve in dotted lines and the corresponding energy constraint in solid lines. We can see that Lemma 1 is correct. According to Theorem 1, with the energy constraint, the dotted lines show the maximum energy a sensor can consume if we set the transmission power as Eq. (18) (apply the water-filling strategy). For different energy-harvesting powers  $P_h$  shown in Fig. 3, the corresponding energy depletion times  $t_0$  are identical owing to the equality of the initial energy  $B_0$ . It means that  $t_0$  is only determined by  $B_0$  and is independent of  $P_h$ , as discussed in subsection 3.2.

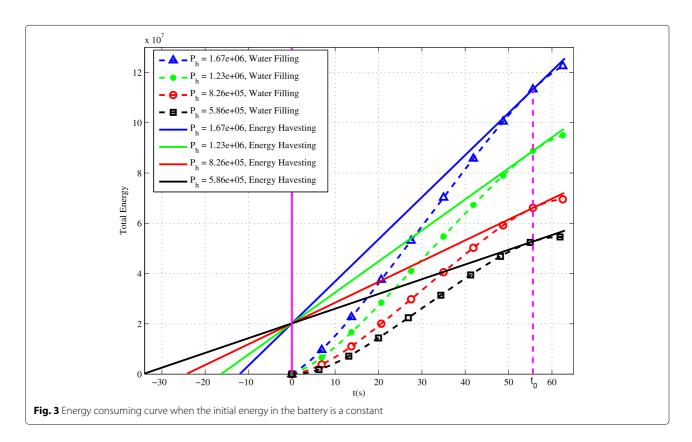
Besides, Fig. 4 illustrates the energy consulting curve and the corresponding energy constraint when the initial energy  $B_0$  varies. When  $B_0$  equals  $5.79 \times 10^5$ ,  $2.02 \times 10^7$ , and  $4.58 \times 10^7$ , respectively,  $t_0$  increases as shown in Fig. 4,

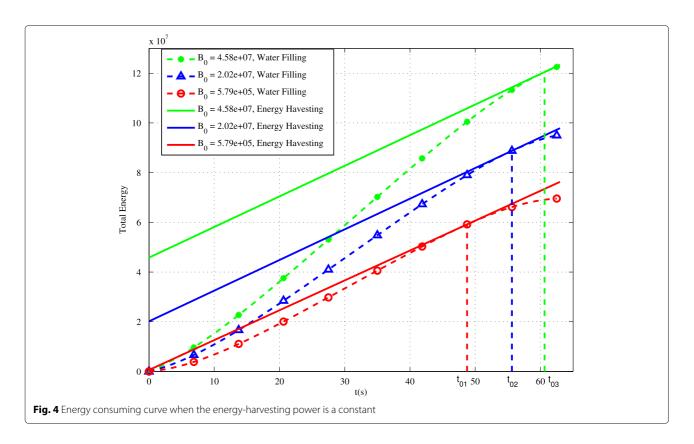
where  $t_{01} < t_{02} < t_{03}$ . It is shown that the energy consumed time  $t_0$  increases with the initial energy  $B_0$ . The simulation result is consistent with Lemma 3.

Moreover, if we set the transmission power as Eq. (18), the energy depletion time  $t_0$  is exactly the end of the timeslot 62.5 s. Under this condition, we obtain the transmission power as shown in Fig. 5. The green dotted lines represent the noise power N(t) and the blue solid line represents the power  $P_s(t)+N(t)$ , which is received at the moving agent. The noise power is a bowl-like curve. Inside this bowl, the area of bright pink is  $\int_0^{2L_0} P_{\rm fill}(t) dt$  as claimed in Theorem 2.

Theorem 2 says the static sensor should at least harvest  $\int_0^{\frac{2L_0}{\nu}} P_{\mathrm{fill}}(t) dt$  energy before triggering a transmission. The value  $\int_0^{\frac{2L_0}{\nu}} P_{\mathrm{fill}}(t) dt$  has an interesting physical meaning as shown in Fig. 5. It suggests that the sensor should at least fill the noise bowl before a transmission is triggered so as to achieve the maximum channel service. Thus, for a certain mobile channel, we can accurately obtain the least energy a sensor should harvest before transmission starts. Just fill the noise bowl, then the transmission can be triggered.

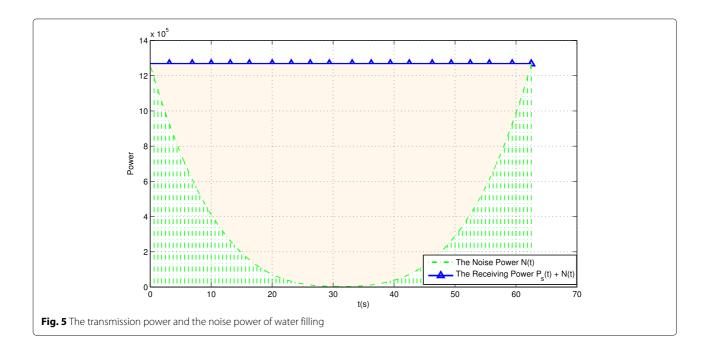
Overall, Figs. 3 and 4 are not the optimum strategy for the energy that shows a small surplus at the end of the timeslot. On the contrary, Fig. 6 simulates the optimum

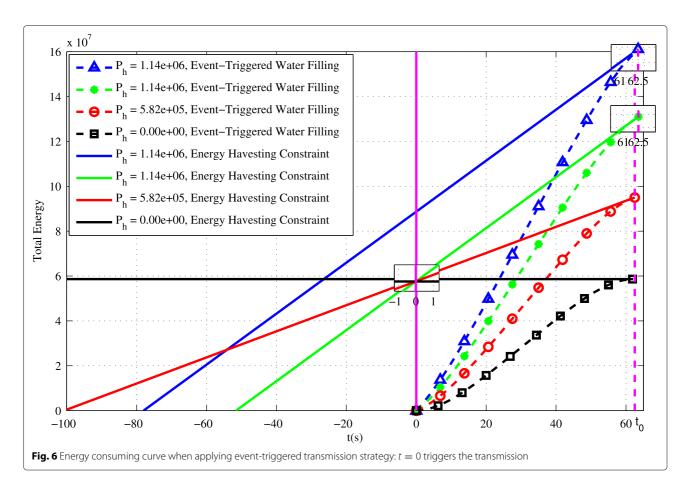




transmission strategy, event-triggered strategy, proposed in subsection 3.3. The dotted lines show that under this strategy, the energy is depleted at the end of the transmission session. From Fig. 6, it is seen that the intersection of three energy constraint solid lines is the minimum

initial energy  $B_0 = \int_0^{\frac{2L_0}{\nu}} P_{\rm fill}(t) dt = 5.86 \times 10^7$ . Thus, the energy constraint is the minimum such that at the end of the timeslot, the transmission power is the same as the energy-harvesting power. As the green, the red, and the black curves shown in Fig. 6, when t = 62.5s,





the solid line is the tangent line of the dotted curve if  $B_0=5.86\times 10^7$ . Conversely, the blue solid line has an initial energy  $B_0=8.87\times 10^7$ , which is bigger than the minimum initial energy  $\int_0^{\frac{2L_0}{\nu}} P_{\rm fill}(t) dt = 5.86\times 10^7$ . Although the energy is depleted at the end of the service session t=62.5 s, the blue line is not the tangent line of the energy consuming curve, which is different from the green, the red, and the black one. This can be seen clearly in Fig. 6 if we enlarge the intersection neighborhood.

Last but not the least, we plot Fig. 7 to show the channel service of transmission strategies. Compared with the hasty transmission, where the PA is constant, our event-triggered policy increases over 10% in terms of channel service.

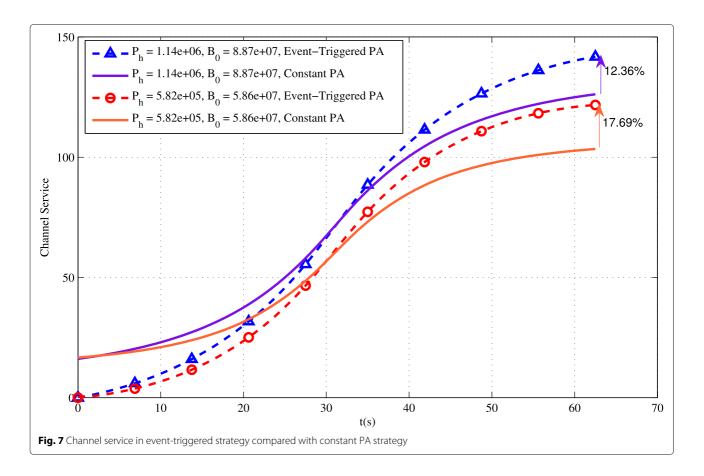
## 5 Conclusions

In this paper, we study the optimum transmission PA in a mobile point-to-point communication system. We first derive the theoretic PA solution under constant harvesting power. Then we propose an event-triggered transmission strategy to achieve the maximum channel service. Two conditions should be satisfied: one is to transmit information throughout the whole service session, and the other is to deplete the energy at the end of the timeslot. Before the transmission is triggered, the sensor should first harvest enough energy. After the transmission is triggered, the sensor continues to harvest energy and provide power for transmitting information at the same time. We calculate the harvesting time and the transmission power of water filling. Also, we give the physical interpretation of the initial energy boundary, i.e., filling the noise bowl to the full. In conclusion, the results in this paper suggest a practical event-triggered transmission strategy that the maximum channel service can be achieved for a mobile transmission system driven by an energy-harvesting sensor. It may provide more insights to the sensor operating design for environment information collecting systems.

# Appendix A Proof of Theorem 1

*Proof* As shown in Fig. 8, there are three cases when setting  $P_s(t) = \left[\frac{W}{\lambda} - N(t)\right]^+$ . Case 1 can be represented as

$$\forall t \in \left(0, \frac{2L_0}{\nu}\right), P_s(t) > 0, \tag{29}$$



while in cases 2 and 3, we have

$$\exists t \in \left(0, \frac{2L_0}{\nu}\right), \ P_s(t) = 0, \tag{30}$$

where  $P_s(t)$  is given by (38) in details. In fact, case 2 degrades to problem (17) as discussed in subsection 3.2. Apparently, when the energy causality constraint is a constant, the corresponding energy-harvesting line is a tangent line of the energy consuming curve. Therefore, in case 2, Theorem 1 is certain to be true.

On the other hand, case 3 is similar to case 1. The only difference is the time interval of transmission. In case 3, the transmission procedure  $T_s$  is just shortened from  $\left[0,\frac{2L_0}{\nu}\right)$  to  $\left[t_b,\frac{2L_0}{\nu}-t_b\right)$ , where  $t_b$  is the time when transmission is triggered, and the analysis is the same with case 1.

So, if we can prove Theorem 1 in case 1, the proof of Theorem 1 will be completed. The following proof is based on condition (29).

First of all, we prove the necessity which means the maximum channel service will lead to the tangent line. We start with the proof of the unicity of tangent line pass-

ing through a certain point  $(0, B_0)$ . Taking the second derivative of  $E_s(t)$ , we get

$$\frac{d^2 E_s(t)}{dt^2} = \frac{d^2 \left[ \int_0^t \left[ \frac{W}{\lambda} - N(\tau) \right] d\tau \right]}{dt^2} = -\frac{dN(t)}{dt}.$$
 (31)

Thus,  $E_s(t)$  is convex when  $t \leq \frac{L_0}{\nu}$  and concave when  $t > \frac{L_0}{\nu}$ . Also,  $E_s(t)$  monotonically increases with time t. Therefore, there is only one line which is tangent to  $E_s(t)$  at  $(0, B_0)$  when  $B_0 \geq 0$ .

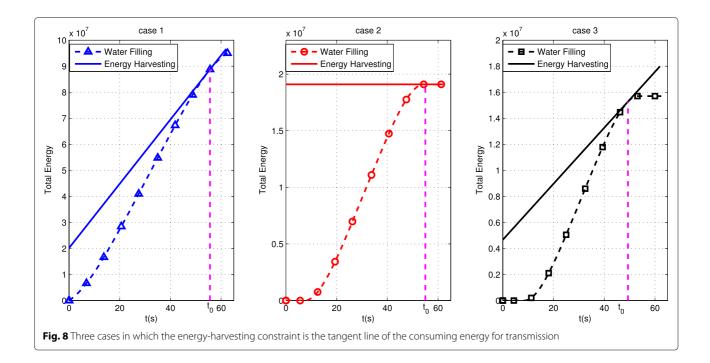
Next, we will prove if the the maximum channel service is achieved,  $B(t) = P_h t + B_0$  must be the tangent line of  $E_s(t) = \int_0^t P_s(\tau) d\tau$ .

Since the energy consumed in transmission is

$$E_s(t) = \int_0^t P_s(\tau) d\tau = \int_0^t \left[ \frac{W}{\lambda} - N(\tau) \right] d\tau,$$

 $E_s(t)$  decreases with  $\lambda$ ,  $\forall t \in \left[0, \frac{2L_0}{\nu}\right]$ .

Suppose  $\tilde{\lambda}$  exits, where  $\tilde{\lambda} < \tilde{\lambda}$  with corresponding transmission energy  $\tilde{E}_s(t) > E_s(t), \forall t \in \left[0, \frac{2L_0}{\nu}\right]$ . Under the conditions that  $B(t) = P_h t + B_0$  is the tangent line of  $E_s(t)$  and the tangent point is  $(t_0, E_0), \tilde{E}_s(t_0) > E_s(t_0)$  and



 $E_s(t_0) = B(t_0) = E_0$  must hold, which means  $B(t_0) - \tilde{E}_s(t_0) < 0$ . So the strategy

$$P_s(t) = \left\lceil \frac{W}{\tilde{\lambda}} - N(t) \right\rceil^+ \tag{32}$$

is not feasible for problem (10).

On the other hand, if  $\tilde{\lambda} > \lambda$ , then  $\tilde{E}_s(t) < E_s(t)$ . Apparently,  $\tilde{\lambda}$  is not desirable because the less transmission energy we have, the less channel service we can get. Apparently, the Lagrangian multiplier  $\tilde{\lambda} > \lambda$  is not optimal.

As a consequence, there is no  $\tilde{\lambda} \neq \lambda$  that can augment the channel service. For  $\lambda 1 \neq \lambda 2$ , the corresponding transmission power  $P_{s1}(t) \neq P_{s2}(t)$ . When the harvesting power and the initial energy in the battery are confirmed, we have the unique water-filling power allocation that can achieve the maximum channel service. Under this PA,  $B(t) = P_h t + B_0$  is the tangent line of  $E_s(t) = \int_0^{2L_0} P_s(\tau) d\tau$ .

Finally, we prove the sufficiency. That is to say, the tangent line will lead to the maximum channel service. As stated above, when  $\tilde{\lambda} > \lambda$ ,  $\tilde{E}_s(t)$  satisfies  $\tilde{E}_s(t) < E_s(t)$ . So B(t) and  $\tilde{E}_s(t)$  do not intersect. On the other hand, when  $\tilde{\lambda} < \lambda$ ,  $\tilde{E}_s(t)$  satisfies  $\tilde{E}_s(t) > E_s(t)$  and the strategy is not feasible. Neither case can achieve the maximum channel service as mentioned above.

## **B Proof of Lemma 1**

*Proof* Given Eq. (18), Theorem 1 illustrates that the energy-harvesting curve  $B(t) = P_h t + B_0$  must be the tangent line to the energy consuming curve  $E_s(t) =$ 

 $\int_0^{\frac{2L_0}{v}} P_s(\tau) d\tau$  at the moment when energy is depleted. Suppose the battery-empty point is  $(t_0, E_0)$  and  $P_s(t)$  satisfies  $\forall t \in \left(0, \frac{2L_0}{v}\right)$ ,  $P_s(t) > 0$ . In accordance with the properties of tangent line, we have

$$E_0 = E_s(t_0) = B(t_0),$$
 (33)

$$P_{\mathcal{S}}(t_0) = P_h. \tag{34}$$

When  $t = t_0$ , the energy constraint in the battery and the energy expended by transmission respectively can be presented as

$$B(t_0) = P_h t_0 + B_0 = P_s(t_0) t_0 + B_0$$

$$= \left(\frac{W}{\lambda} - N(t_0)\right) t_0 + B_0$$
(35)

$$E_{s}(t_{0}) = \int_{0}^{t_{0}} P_{s}(t)dt = \int_{0}^{t_{0}} \left(\frac{W}{\lambda} - N(t)\right)dt$$
$$= \frac{W}{\lambda}t_{0} - \int_{0}^{t_{0}} N(t)dt$$
(36)

From (33), (35), and (36), we can get

$$B_0 = N(t_0)t_0 - \int_0^{t_0} N(t)dt \tag{37}$$

which indicates that  $t_0$  is only determined by  $B_0$  but has nothing to do with  $P_h$ .

From the above, we have proved the sufficient condition in Lemma 1. Next, we will prove the necessary condition.

If  $\exists t \in (0, \frac{2L_0}{\nu})$ ,  $P_s(t) = 0$ , let  $t_b$  be the time when the sensor begins to transmit information, which means

$$P_{s}(t) = \begin{cases} 0, & t \in [0, t_{b}) \\ \frac{W}{\lambda} - N(t), & t \in \left[t_{b}, \frac{2L_{0}}{\nu} - t_{b}\right] \\ 0, & t \in \left(\frac{2L_{0}}{\nu} - t_{b}, \frac{2L_{0}}{\nu}\right) \end{cases}$$
(38)

Then, the relation between the initial energy in the battery  $B_0$  and the depletion time  $t_0$  is

$$B_0 = N(t_0)t_0 - \int_{t_h}^{t_0} N(t)dt - \frac{W}{\lambda}t_b.$$
 (39)

It is shown that  $t_0$  is related to both  $t_b$  and  $B_0$ , where  $t_b$  is determined by  $P_h$ . Thus, if  $t_0$  is uniquely determined by  $B_0$ ,  $\forall t \in (0, \frac{2L_0}{u})$ ,  $P_s(t) > 0$  must be satisfied.

#### C Proof of Theorem 2

*Proof* We consider Eq. (18) as the PA for problem (10). Lemma 3 has proved that if

$$B_0 \leq \int_0^{\frac{2L_0}{\nu}} P_{\text{fill}}(t) dt,$$

the energy depletion time  $t_0$  satisfies  $t_0 < \frac{2L_0}{\nu}$ . When  $t_0 < \frac{2L_0}{\nu}$ , according to Eq. (31), the relation between the transmission power and the harvesting power is

$$P_s(t) < P_h, \ t \in \left(t_0, \frac{2L_0}{\nu}\right].$$
 (40)

Thus, at the end of the timeslot, the energy left in the battery  $E_{\text{left}}$  is

$$E_{\text{left}} = \int_{t_0}^{\frac{2L_0}{\nu}} [P_h - P_s(t)] dt > 0.$$
 (41)

So if energy is exhausted at the end of the timeslot, the initial energy  $B_0$  must satisfy inequality (22). The converse is also true. If  $B_0$  satisfies inequality (22), according to Lemmas 1 and 3, no matter what value  $P_h$  is, the time when energy is exhausted satisfies  $t_0 \geq \frac{2L_0}{\nu}$ .

To sum up, we have proved that inequality (22) is the sufficient and necessary condition for  $t_0 \ge \frac{2L_0}{\nu}$ . Meanwhile,  $t_0 \in \left[0, \frac{2L_0}{\nu}\right]$ , so

$$t_0 = \frac{2L_0}{\nu}. (42)$$

Thus, Theorem 2 is equivalent to the statement that the PA (18) with a proper  $\lambda > 0$  can be optimal for problem (10) if and only if energy should be exhausted at the end of the timeslot.

The proof of this statement is intuitive. If the energy is not exhausted at the end of the timeslot, the remaining energy can be used to transmit information, which indicates that the current PA cannot achieve the maximum channel service. On the other hand, if we already know that the energy is depleted at the end of the timeslot, the

harvesting energy line and the transmission energy curve intersects at  $t = \frac{2L_0}{v}$ , which means

$$B\left(\frac{2L_0}{\nu}\right) = E_s\left(\frac{2L_0}{\nu}\right), \ P_h\left(\frac{2L_0}{\nu}\right) < P_s\left(\frac{2L_0}{\nu}\right). \ (43)$$

Since the PA is feasible, the water-filling strategy can absolutely achieve the maximum throughput as subsection 3.2 discussed.

#### Acknowledgements

The authors would like to thank Kun Chen from Tsinghua University for his valuable advice on this paper and Dr. Chen Zhengchuan for fruitful discussions.

#### Funding

None.

#### Authors' contributions

YT has finished all the system modeling, analysis, simulation, and drafting the article. SL has helped revise the manuscript. Professor PF has given critical revision of the article and has helped revise the manuscript. All authors read and approved the final manuscript.

#### **Competing interests**

The authors declare that they have no competing interests.

#### **Publisher's Note**

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 31 July 2017 Accepted: 27 September 2017 Published online: 14 November 2017

#### References

- S Ulukus, A Yener, E Erkip, O Simeone, M Zorzi, P Grover, K Huang, Energy harvesting wireless communications: a review of recent advances. IEEE J. Sel. Areas Commun. 33(3), 360–81 (2015)
- J Yang, S Ulukus, Optimal packet scheduling in an energy harvesting communication system. IEEE Trans. Commun. 60(1), 220–30 (2012)
- B Devillers, D Gunduz, A general framework for the optimization of energy harvesting communication systems with battery imperfections.
   J. Commun. Netw. 14(2), 130–9 (2012)
- CK Ho, R Zhang, Optimal energy allocation for wireless communications with energy harvesting constraints. IEEE Trans. Sig. Process. 60(9), 4808–18 (2012)
- K Xiong, P Fan, C Zhang, KB Letaief, Wireless information and energy transfer for two-hop non-regenerative mimo-ofdm relay networks. IEEE J. Sel. Areas Commun. 33(8), 1595–611 (2015)
- K Xiong, B Wang, KJR Liu, Rate-energy region of swipt for mimo broadcasting under nonlinear energy harvesting model. IEEE Trans. Wirel. Commun. PP(99), 1–1 (2017)
- X Di, K Xiong, P Fan, HC Yang, Simultaneous wireless information and power transfer in cooperative relay networks with rateless codes. IEEE Trans. Veh. Technol. 66(4), 2981–96 (2017)
- R Jiang, K Xiong, P Fan, Y Zhang, Z Zhong, Optimal design of SWIPT systems with multiple heterogeneous users under non-linear energy harvesting model. IEEE Access. PP(99), 1–1 (2017)
- K Tutuncuoglu, A Yener, Optimum transmission policies for battery limited energy harvesting nodes. IEEE Trans. Wirel. Commun. 11(3), 1180–9 (2012)
- O Ozel, K Tutuncuoglu, J Yang, S Ulukus, A Yener, Transmission with energy harvesting nodes in fading wireless channels: optimal policies. IEEE J. Sel. Areas Commun. 29(8), 1732–43 (2011)
- K Xiong, C Chen, G Qu, P Fan, K Letaief, Group cooperation with optimal resource allocation in wireless powered communication networks. IEEE Trans. Wirel. Commun. PP(99), 1–1 (2017)
- N Michelusi, K Stamatiou, M Zorzi, Transmission policies for energy harvesting sensors with time-correlated energy supply. IEEE Trans. Commun. 61(7), 2988–3001 (2013)

- N Michelusi, K Stamatiou, M Zorzi, in Proceedings of IEEE Information Theory and Applications Workshop. On optimal transmission policies for energy harvesting devices, (2012), pp. 249–254
- V Sharma, U Mukherji, V Joseph, S Gupta, Optimal energy management policies for energy harvesting sensor nodes. IEEE Trans. Wirel. Commun. 9(4), 1326–36 (2008)
- J Lei, R Yates, L Greenstein, A generic model for optimizing single-hop transmission policy of replenishable sensors. IEEE Trans. Wirel. Commun. 8(2), 547–51 (2009)
- J Liu, K Xiong, P Fan, Z Zhong, Rf energy harvesting wireless powered sensor networks for smart cities. IEEE Access. PP(99), 1–1 (2017)
- L Xie, Y Shi, YT Hou, W Lou, in *In Proceedings of IEEE INFOCOM*. On renewable sensor networks with wireless energy transfer: The multi-node case, (2012), pp. 1350–8
- S He, J Chen, Y Sun, DKY Yau, NK Yip, On optimal information capture by energy-constrained mobile sensors. IEEE Trans. Veh. Technol. 59(5), 2472–84 (2009)
- C Zhang, P Fan, Y Dong, K Xiong, Service-based highspeed railway base station arrangement. Wirel. Commun. Mob. Comput. 15(13), 1681–94 (2013)
- T Li, P Fan, Z Chen, KB Letaief, Optimum transmission policies for energy harvesting sensor networks powered by a mobile control center. IEEE Trans. Wirel. Commun. 15(9), 6132–45 (2016)
- 21. Y Dong, P Fan, KB Letaief, High-speed railway wireless communications: efficiency versus fairness. IEEE Trans. Veh. Technol. **63**(2), 925–30 (2014)
- Y Tan, Z Chen, P Fan, in Proceedings of Asia-Pacific Conference on Communications. Optimal Event-Triggered Strategy for Energy Harvesting Mobile Transmission, (2017), pp. 1–6
- J Huang, D Shi, T Chen, Event-triggered state estimation with an energy harvesting sensor. IEEE Trans. Autom. Control. PP(99), 1–1
- 24. AA El Gamal, A Abbas, YH Kim, *Network Information Theory*. (Cambridge University Press, 2011)

# Submit your manuscript to a SpringerOpen journal and benefit from:

- ► Convenient online submission
- ► Rigorous peer review
- ▶ Open access: articles freely available online
- ► High visibility within the field
- ► Retaining the copyright to your article

Submit your next manuscript at ▶ springeropen.com