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# Research of combination forecasting model based on improved analytic hierarchy process

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## Abstract

The weighting method of the traditional fixed combination forecasting model is the only criterion considered to improve accuracy, which has some limitations. In order to improve the comprehensive prediction performance of the combined model, hierarchical structure of the combined model by selecting some parameters which can reflect the performance of the model (including prediction accuracy, robustness, sensitivity, and the amount of fitting data) is established and a kind of multiple factor and multiple criteria weighting method of combination forecasting model is put forward. Based on SVR model, GM (1, 1) model, and ARIMA model, a combination forecasting model based on Improved Analytic Hierarchy Process (AHP) is constructed and applied to a foundation pit. The experimental results show that the combined forecasting model based on improved AHP are better than the single model in precision and robustness; it also has good effect in sensitivity, which has more comprehensive prediction performance than the single models, and has good engineering and practical value.

**Keywords:** SVR model, GM (1, 1) model, ARIMA model, Improved analytic hierarchy process, Combination forecasting model

## 1 Introduction

Because of the complexity of the deformation and the limitations of various forecasting model, it is a trend [1] to forecast deformation accurately by using effective information of multiple models. So the combination model of weight problem also will become the research focus. Pengpeng et al. [2] established a various weight combination based on linear regression prediction model and gray model GM (1,1); Shaofeng et al. [3] established a combination prediction model based on entropy weight method; Caiyun et al. [4] established five types of deformation parallel combination forecasting models that include sub-optimal weight, optimal weight, gray comprehensive correlation degree weight, entropy weight, and neural network. And the characteristics of weight seeking under the five kinds of constraints are discussed. But for now, most of the research work to determine the weights of each single model is based on minimizing the sum of square errors or sum of absolute values of error [5]. So there are defects that are not considered in the advantages and disadvantages of a single model in all aspects of performance: cannot make full use of its information and simple improvement in the precision, and the comprehensive prediction performance is not optimal, for example, when there are outliers, model may fail. Paper [6] also pointed out that when choosing prediction models, we should not only consider whether the prediction results of the model can reflect the complexity of the environment, but also have certain accuracy, and we should consider whether the model can be accepted by users and other related factors. In view of this situation, the evaluation system for the performance of each model is constructed, and the weight of each model is determined by combining with the improved AHP (Analytic Hierarchy Process, AHP for short). Taking the data of a foundation pit in Guangzhou as an example, the chosen three single models include SVR, GM (1,1), and ARIMA. Then, the combined forecasting model based on the improved AHP is established and the application research is carried out.

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## 2 Brief introduction of basic model

### 2.1 SVR model

The full name of SVR is support vector regression; it was put forward by Vapnik in 1990s, and it is a new statistical method [7], which can deal with many problems such as regression and pattern recognition. The basic idea is to use the known sample data to obtain an optimal fitting function. The modeling process [8] is:

(1) Giving a training set:

$$T = \{(x_1, y_1), \cdots (x_l, y_l)\} \in (R^n \times Y)^l \quad (1)$$
$$x_i \in R^n, y_i \in Y = R, i = 1, \cdots, l$$

(2) Mapping sample input to high-dimensional space by nonlinear mapping and constructing the function of SVR:

$$y(x) = \boldsymbol{\omega}^T \boldsymbol{\phi}(x) + b \quad (2)$$

"$\omega$" is a weight vector, and "$b$" is a deviation.

(3) Turn it into the following optimization problem:

$$\min_{(\omega, b, \xi, \xi^*)} \frac{1}{2}\|\omega\|^2 + c\sum_{i=0}^{n}(\xi_i + \xi_i^*) \quad (3)$$

$$s.t \quad \begin{cases} y_i - \omega \cdot \phi(x_i) - b \le \varepsilon + \xi_i \\ \omega \cdot \phi(x_i) + b - y_i \le \varepsilon + \xi_i^* \\ \xi_i \ge 0, \quad \xi_i^* \ge 0 \end{cases} \quad (4)$$

"$c$" is the penalty parameter, and "$\xi_i \ge 0$" is the relaxation factor.

(4) The function model of SVR can be solved by using Lagrange function and KKT optimization condition:

$$y(x) = \sum_{i=1}^{l}(a_i - a_i^*)K(x, x_i) + b \quad (5)$$

"$a_i$" is the Lagrange multiplier, and the "$K(x, x_i)$" is a kernel function.

(5) The deformation prediction model needs strong fitting ability. Therefore, the Gauss radial basis function is used as the kernel function of SVR:

$$K(x, x_i) = \exp\left(-\frac{\|x - x_i\|_2^2}{2\sigma^2}\right) \quad (6)$$

### 2.2 GM (1, 1) model

The gray model is first proposed by Professor Deng Julong [9], which is mainly used to solve the prediction problem under the condition of poor information. In the prediction of deformation monitoring, a differential equation of first order and one variable is generally adopted, which is called GM (1,1), the modeling process [10, 11] is:

(1) Set the original deformation monitoring sequence as:

$$x^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \Lambda, x^{(0)}(n)\right\} \quad (7)$$

(2) A cumulative addition:

$$\begin{gathered} x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \Lambda, x^{(1)}(n)\right\} \\ x^{(1)}(t) = \sum_{i=1}^{t} x^{(0)}(i) \\ t = 1, 2, \Lambda, n \end{gathered} \quad (8)$$

(3) The establishment of first-order differential equation:

$$\frac{d_x^{(1)}(t)}{dt} + ax^{(1)}(t) = b \quad (9)$$

(4) Matrix form is:

$$y_N = Bf \quad (10)$$

(5) The "$a$" and "$b$" can be obtained by the least square method:

$$\hat{x}^{(1)}(t+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-at} + \frac{b}{a} \quad (11)$$

(6) The solution of first-order differential equation:

$$\hat{x}^{(1)}(t+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-at} + \frac{b}{a} \tag{12}$$

(7) Data restore:

$$\begin{aligned} &\hat{x}^{(0)}(t+1) = \hat{x}^{(1)}t + 1 - \hat{x}^{(1)}(t) \\ or &\hat{x}^{(0)}(t+1) = (1-e^{a})\left(x^{(0)}(1) - \frac{b}{a}\right)e^{-at} \end{aligned} \tag{13}$$

In the above formula, "$a$, $b$" is a gray parameter, and "$t$" is a time series.

### 2.3 ARIMA model

The full name of ARIMA is auto-regressive integrated moving average model; it is a famous time series prediction method which was put forward by Box and Jenkins in early 1970s. The ARIMA model is expressed as:

$$\begin{aligned} x_t = {} & \phi_1 x_{t-1} + \phi_2 x_{t-2} + \Lambda + \phi_n x_{t-n} - \theta_1 a_{t-1} - \theta_2 a_{t-2} \\ & - \Lambda - \theta_m a_{t-m} + a_t \end{aligned} \tag{14}$$

In the above formula, $a_t \sim N(0, {\sigma_0}^2)$, $\phi_i(i = 1, 2, \Lambda, n)$ is called the autoregressive parameter, and $n$ is the order of the model.

## 3 Combined model and methodology

### 3.1 Improved AHP

AHP is a combination of qualitative and quantitative, systematic, and hierarchical analysis method. The basic idea of traditional AHP [12] is as follows: each factor of analysis system is divided into several levels according to different properties, then the judgment matrix is constructed by paired comparison method on the same level, next the maximum characteristic solution and the corresponding eigenvector of each judgment matrix are calculated and the consistency check is done. Finally, the weight vector is obtained. This paper makes the following improvements in view of the shortcomings of the traditional AHP.

(1) Because the accuracy requirement of the "1–9" scale in the traditional AHP is very high for the accuracy of the judgment of the importance degree, so it is difficult to grasp and improve the human-centered view. Therefore, the "0–2" three scale method is used to replace the "1–9" scale.
(2) The comparison matrix constructed by expert evaluation of traditional AHP, considering that even the same model will have different effects in different projects, this paper adopts the "backward method" to construct the comparison matrix; first, the single model is used to predict, and then the comparison matrix is constructed according to the prediction results.
(3) In this paper, the concept of optimal transfer matrix in document [13] is introduced, which avoids repeated checking consistency in traditional AHP, reduces computation, and simplifies the model.

The detailed steps of the improved AHP are as follows:

(1) Establishing a hierarchical structure model. The top is the target layer, the middle is the index layer, and the bottom is the program layer.
(2) Constructing a comparison matrix by "0–2" three scale method. The comparison matrix is constructed according to the prediction results of the single model.

$$A_{ij} = \begin{bmatrix} a_{11} & a_{12} & \Lambda & a_{1n} \\ a_{21} & a_{22} & \Lambda & a_{2n} \\ M & M & O & M \\ a_{n1} & a_{n2} & \Lambda & a_{nn} \end{bmatrix} \tag{15}$$

$$a_{ij} = \begin{cases} 0 \text{ The importance of } i \text{ factoris not as good as } j \text{ factor} \\ 1 \text{ The importance of } i \text{ factoris equal to } j \text{ factor} \\ 2 \text{ The importance of } j \text{ factoris not as good as } i \text{ factor} \end{cases}$$

(3) Calculation of importance ranking index "$r_{ij}$":

$$r_i = \sum_{j=1}^{n} A_{ij} \tag{16}$$

(4) Establishing judgment matrix "$B$":

$$\begin{aligned} &B = \begin{bmatrix} b_{11} & b_{12} & \Lambda & b_{1n} \\ b_{12} & b_{22} & \Lambda & b_{2n} \\ M & M & O & M \\ b_{n1} & b_{n2} & \Lambda & b_{nn} \end{bmatrix} \\ &b_{ij} = \begin{cases} \dfrac{r_i - r_j}{r_{\max} - r_{\min}}(k-1) & r_i \langle r_j \\ \left[\dfrac{r_j - r_i}{r_{\max} - r_{\min}}(k-1) + 1\right]^{-1}, r_i \langle r_j \end{cases} \\ &r_{\max} = \max(r_i), r_{\min}(r_i), k_m = \frac{r_{\max}}{r_{\min}} \end{aligned} \tag{17}$$

(5) Finding the optimal transfer matrix "$C$":

$$c_{ij} = \frac{1}{n}\sum_{l=1}^{n}\left(\lg(b_{il}) - 1g(b_{jl})\right) \tag{18}$$

(6) Finding the quasi optimal uniform matrix "$D$":

$$d_{ij} = 10^{c_{ij}} \tag{19}$$

(7) Finding the eigenvector “$w$” corresponding to the maximum eigenvalue of “$D$,” and obtaining weight vector by normalized “$w$”:

$$w' = \frac{w - w_{\min}}{w_{\max} - w_{\min}} \tag{20}$$

In the formula (9), “$w$” is eigenvector, and “$w^{'}$” is weight vector.

### 3.2 Combined model constructing

The basic idea to construct the combined forecasting model based on the improved AHP is as follows: first, selecting the performance evaluation index of the model then weighting for each model by improved AHP, so a combined prediction model is constructed, and the steps of the modeling are as follows:

(1) There is $m$ kinds of single prediction model, and $y_i$ is the predictive value of the $i$ model ($i = 1, 2, \Lambda, m$);
(2) Combined with the literature and the actual situation, the evaluation index of the model is selected as follows: prediction accuracy, robustness, sensitivity, and the amount of data needed to be fitted. A hierarchical structure model is set up as shown in Fig. 1.

Robust [14]is also known as reliability; different domains have different definitions. In this paper, it is defined as the ability of the model to resist the gross error. Sensitivity [15]is the extent to which the change of information affects the ranking results of the scheme. In this article, it is defined as the extent to which the model can reflect the subtle changes in the input data.

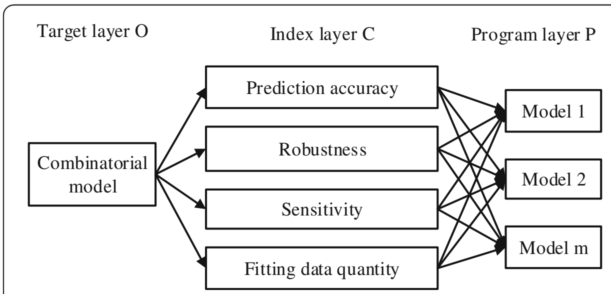


**Fig. 1** Hierarchical structure diagram of combined model. As a hierarchical structure diagram, it consists of three layers: target layer, index layer, and program layer. The program layer contains a variety of intelligent model types, and which model should be chosen by the actual needs of the project. The index layer contains a variety of evaluation indexes of the single model: prediction accuracy, robustness, sensitivity, and the amount of data needed to be fitted. These selected indexes can reflect the performance of the model

(3) The comparison matrix is constructed according to the prediction results of each single model.
(4) The weight of each single model is determined by the improved AHP.

$$w = (w_1, w_2, \Lambda, w_m) \tag{21}$$

(5) Combination model construction.

$$f = \sum_{i=1}^{m} w_i \cdot y_i \tag{22}$$

(6) Rolling prediction: first, input $m$ period training sample to predict the $m$ + 1 deformation data, and then add the predicted $m$ + 1 data to the training sample to form a new training sample (keeping the number of training sample is constant), continue to predict the $m$ + 2 data, and so on, until all the samples to be predicted are drawn.
(7) Performance evaluation of combined model.

A combined prediction model building flow chart based on improved AHP is shown in Fig. 2.

## 4 Evaluation index

In order to comprehensively evaluate the effect of the combined model, the effects of each model on deformation prediction of foundation pit were evaluated by root mean square error, deviation rate, and response rate.

(1) Root mean square error (RMSE)
Evaluating the prediction accuracy of each model by the RMSE, there is the worse prediction effect with the larger RMSE and the better prediction effect with the smaller RMSE.

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} (y_t - \hat{y}_t)^2} \tag{23}$$

(2) Deviation rate (DR)
Evaluating the robustness of the model by DR, the meaning of DR is the degree of deviation between the predicted values when the fitting data contains

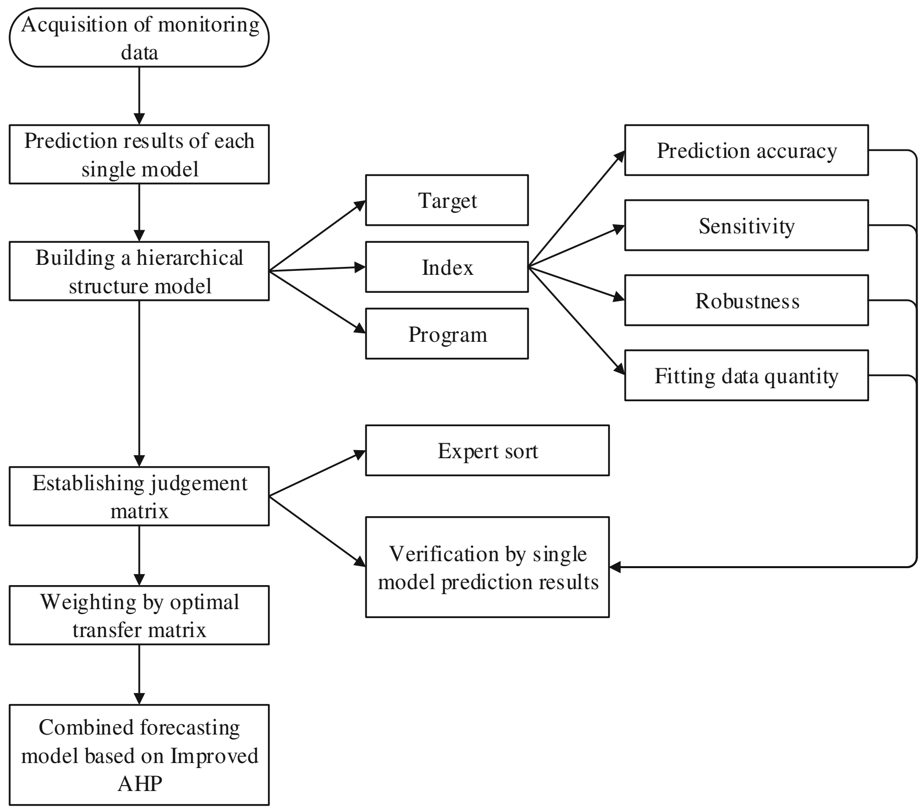


**Fig. 2** Flowchart of construction of the combined model. In order to make readers understand the whole structure of the combined model more clearly, there is a flowchart from data preparation to determine the weight of each single model. The detailed solution steps can be seen in Section 3

gross error and the original predicted values when the gross error is not included in the fitting data; there is the worse robustness with the larger DR and the better robustness with the smaller DR.

$$\mathrm{DR} = \frac{\text{predicted value contains gross error}-\text{original predicted values}}{\text{original predicted values}} \times 100\% \tag{24}$$

(3) Response rate (RR)

Evaluating the sensitivity of the model by RR, the meaning of RR is the rate of change between the predicted values of each model and the original predicted value when the fitting data is slightly disturbed. There is the worse sensitivity with the smaller RR and the better sensitivity with the larger RR.

$$\mathrm{RR} = \frac{\text{predicted value with slightly disturbed}-\text{original predicted value}}{\text{original predicted value}} \times 100\% \tag{25}$$

## 5 Simulation results and discussion

Taking the monitoring data of a foundation pit project in Guangzhou [16], which are shown in Table 1, as an example, it has 21 sets of settlement data; in this paper, the first 18 phases are used as model fitting data, and the latter 3 are used as prediction data.

### 5.1 Simulation setting and results

Selecting the SVR, GM (1,1), and ARIMA as the basic models, predictive value A is predicted by three single models with original fitting data, calculating the prediction RMSE of each model; predictive value B is predicted by three single models when the fitting data contains

**Table 1** Foundation pit monitoring data

| Periods | Observations/ mm | Periods | Observations/ mm | Periods | Observations/ mm |
|---|---|---|---|---|---|
| 1 | 3.34 | 8 | 4.00 | 15 | 4.27 |
| 2 | 3.36 | 9 | 4.10 | 16 | 4.31 |
| 3 | 3.42 | 10 | 4.13 | 17 | 4.32 |
| 4 | 3.72 | 11 | 4.19 | 18 | 4.35 |
| 5 | 3.83 | 12 | 4.22 | 19 | 4.41 |
| 6 | 3.90 | 13 | 4.23 | 20 | 4.48 |
| 7 | 3.97 | 14 | 4.25 | 21 | 4.55 |

**Table 2** Prediction results of single model

| Periods | SVR | | | GM (1,1) | | | ARIMA | | |
|---|---|---|---|---|---|---|---|---|---|
| | A/mm | B/mm | C/mm | A/mm | B/mm | C/mm | A/mm | B/mm | C/mm |
| 19 | 4.449 | 4.144 | 4.254 | 4.544 | 5.076 | 4.487 | 4.38 | 4.207 | 4.371 |
| 20 | 4.454 | 4.130 | 4.159 | 4.605 | 5.161 | 4.542 | 4.434 | 4.229 | 4.417 |
| 21 | 4.453 | 4.116 | 4.073 | 4.668 | 5.248 | 4.597 | 4.498 | 4.272 | 4.478 |

gross error (fifth and tenth observations were changed to 6 and 8 mm respectively), calculating the prediction DR of each model; predictive value C is predicted by three single models when the fitting data is slightly disturbed (fifth and tenth observations were adjusted to 4 and 4 mm respectively), calculating the prediction RR of each model. The results are shown in Table 2.

According to Table 2, the hierarchical structure model, the comparison matrix of the program layer to the index layer and the comparison matrix of the index layer to the target layer are constructed respectively. It can obtain the weights of SVR, GM (1,1), and ARIMA which are 0.326, 0.162, and 0.512 respectively by using MATLAB to write an improved AHP program.

So the expression of the combined model is as follows:

$$f = 0.326l_1 + 0.162l_2 + 0.512l_3$$

The standard predictive value A, the robustness predictive value B, and the sensitivity predictive value C predicted by the combined model are shown in Table 3. The RMSE of the predictive value A, the DR of the predictive value B, and the RR of the predictive value C calculated by single model and combination model are shown in Table 4.

## 6 Discussion

According to Table 4, in terms of the prediction accuracy, the minimum residual of the combined model is 0.012, the maximum is 0.039, and the RMSE is + 0.026, which is lower than the other single model, and as shown in Fig. 3, the deformation curve predicted by the combined model is more consistent with the observed deformation curve, which shows that the combined model is more capable of reflecting the deformation trend than the single model. So the prediction accuracy from high to low is as follows: the combination model > ARIMA model > SVR model > GM (1,1) model. In terms of robustness, the minimum DR of the combined model is 2.3, the maximum is 2.93, and the average DR is 2.53, which is lower than all the single models, indicating that the combined model is least affected by gross errors, and as shown in Fig. 4, the predicted deformation curve of the combined model is most close to the actual deformation curve, indicating that the prediction effect of the combined model is the most stable. So the robustness from optimal to inferior is as follows: the combination model > ARIMA model > SVR model > GM (1,1) model. In terms of sensitivity, the average RR of the combined model is 2.51, which is higher than the that of the GM (1,1) model and ARMIA model, indicating that the combined model can also respond to smaller deformation, and as shown in Fig. 5, the prediction curve of the combined model has obvious changes, and its amplitude is only slightly smaller than that of the SVR model. In terms of the required amount of fitting data, the adjustment of the weight of the combined model can weaken the demand for a single model with more data. Therefore, combining the above results, the combined model is better integrated with the advantages of the single model, which can achieve better results in all aspects.

**Table 3** Forecast results of the combined model

| Periods | Observations/ mm | Predictions A/ mm | Predictions B/ mm | Predictions C/ mm |
|---|---|---|---|---|
| 19 | 4.41 | 4.438 | 4.376 | 4.375 |
| 20 | 4.48 | 4.484 | 4.401 | 4.378 |
| 21 | 4.55 | 4.531 | 4.437 | 4.39 |

**Table 4** Comparison of the effect of single-item model and the combined model

| Periods | | 19 | 20 | 21 | RMSE | Average |
|---|---|---|---|---|---|---|
| SVR | Residual/ mm | − 0.039 | 0.026 | 0.097 | ± 0.062 | |
| | DR/% | 6.86 | 7.27 | 7.57 | | 7.23 |
| | RR/% | 4.38 | 6.62 | 8.53 | | 6.51 |
| GM (1,1) | Residual/ mm | − 0.134 | − 0.125 | − 0.118 | ± 0.126 | |
| | DR/% | 11.71 | 12.07 | 12.43 | | 12.07 |
| | RR/% | 1.25 | 1.37 | 1.52 | | 1.38 |
| ARIMA | Residual/ mm | 0.03 | 0.046 | 0.052 | ± 0.044 | |
| | DR/% | 3.95 | 4.62 | 5.02 | | 4.53 |
| | RR/% | 0.21 | 0.38 | 0.44 | | 0.34 |
| Combination model | Residual/ mm | − 0.019 | 0.012 | 0.039 | ± 0.026 | |
| | DR/% | 2.3 | 2.35 | 2.93 | | 2.53 |
| | RR/% | 1.74 | 2.57 | 3.24 | | 2.51 |

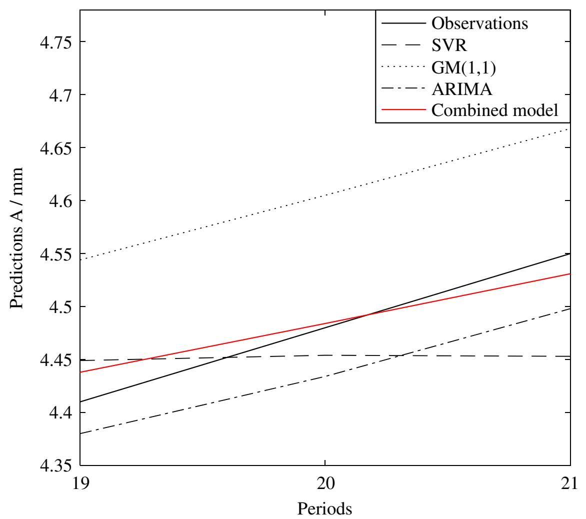


**Fig. 3** Predictions A of each model. Selecting the SVR, GM (1,1), ARIMA as the basic models, predictive value A is predicted by three single models with original fitting data. According to the predictions A of the single model and combined model, and the observations, we connect the respective values through the broken line, which can form the most direct contrast

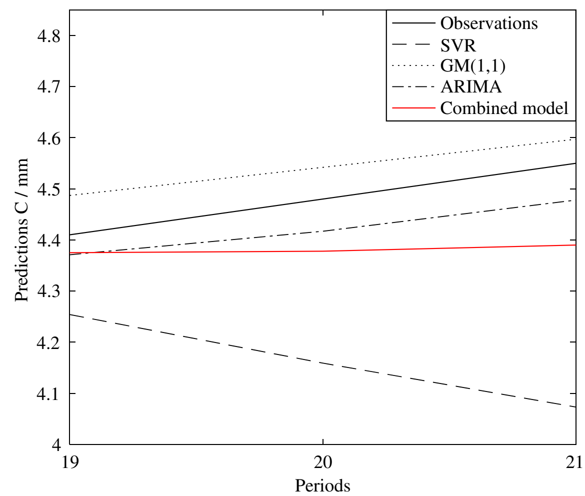


**Fig. 5** Predictions C of each model. Predictive value C is predicted by three single models when the fitting data is slightly disturbed (fifth and tenth observations were adjusted to 4 and 4 mm respectively). According to the predictions C of single model and combined model, and the observations, we connect the respective values through the broken line, which can form the most direct contrast

## 7 Conclusions

Different single models have their own advantages and disadvantages. Combining with each single prediction model in an appropriate way can fully draw the advantages of each single prediction model and avoid its shortcomings, so that the deformation prediction is carried out in a more comprehensive way. This paper selects several indicators that can reflect the performance of the prediction model then solves the weights of each single model by the improved AHP, so a combination model for deformation prediction is built. The application results show that the combined model built in this paper achieves good results in prediction accuracy, robustness, and sensitivity, and the comprehensive performance is better than all single prediction models, thus providing a new method for deformation prediction research.

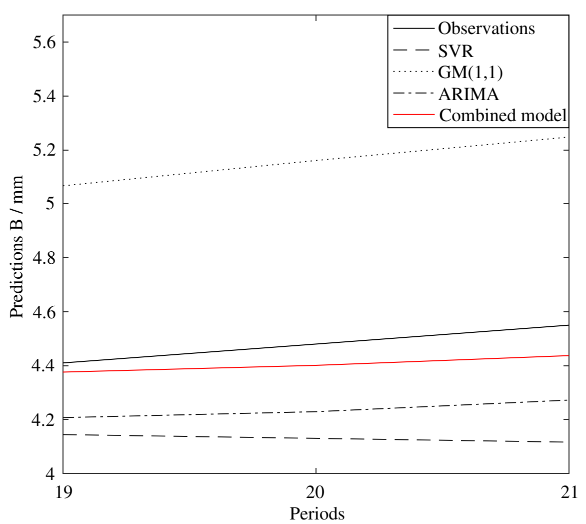


**Fig. 4** Predictions B of each model. Predictive value B is predicted by the three single models and combined model when the fitting data contains gross error (fifth and tenth observations were changed to 6 and 8 mm respectively). According to the predictions B of single model and combined model, and the observations, we connect the respective values through the broken line, which can form the most direct contrast

### Abbreviations

AHP: Analytic Hierarchy Process; ARIMA: Auto-regressive integrated moving average; DR: Deviation rate; RMSE: Root mean square error; RR: Response rate; SVR: Support vector regression


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### Availability of data and materials

The datasets supporting the conclusions of this article were collected from reference [16].

### Authors' contributions

TF is the main writer of this paper. He proposed the new weighting method, deduced the whole process of model building, completed the experience, and analyzed the result. XL put forward some suggestions for improvement. YZ and SL wrote the program of the algorithm. All authors read and approved the final manuscript.

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**Competing interests**
The authors declare that they have no competing interests.

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**References**

1. Li Xiuzhen, Kong Jiming, Wang Chenghua.Application of combined model with optimum weight in prediction of landslide deformation [J].J Nat Dis. (02):53–57 (2008)
2. Xie Pengpeng, Huang Teng, Liu Yang. Application of changeable weight combination forecasting model to dam settlement predicting[J]. Eng Surv Mapp. **24**(04):74–76 (2015)
3. Hao Shaofeng, Fang Yuanmin, Yang Jianwen, et al. Application of combination model of entropy method to landslide deformation prediction[J]. Eng Surv Mapp. **23**(07):62–64 (2014)
4. Gao Caiyun, Cui Ximin, Gao Ning. Study on deformation parallel combination prediction under different restriction criterion[J].J Geodesy Geodynamics. **34**(03):91–94 (2014)
5. Wang Jintao, Guo Guangli, Zhao Ziqiang, et al. A combined model for mining subsidence forecasting based on correlativity[J]. Sci Surv Mapp. **42**(10):1–8 (2017)
6. Wang Liang. Selection of prediction model and its intelligent implementation[C]: scientific decision making and system engineering -- Proceedings of the Sixth Annual Meeting of China Society for Systems Engineering. 7 (1990)
7. Vapnik V N. The Nature of Statistical Learning Theory[M]. (New York: Springer, 1995)
8. Li Ning, Wang Liguan, Jia Mingtao.Rockburst prediction based on rough set theory and support vector machine[J]. J Central South Univ(Sci Technol). **48**(05):1268–1275 (2017)
9. Deng J L.Introduction to grey system theory[J]. J Grey System. **1**(1):1–24 (1989)
10. Wen Hongyan, Zhou Lu, Han Yakun, et al.. Application of GM(1,1) model based on Kalman filter to the subsiding analysis of high-speed railway tunnel[J]. J Geodesy Geodynamics. **4**(01):88–91 (2014)
11. P Zhenbin, Z Chuang, P Wenxiang, et al., Different structure methods and application of background value in GM(1,1) model. J Northeastern Univ (Natural Science) **38**(06), 869–873 (2017)
12. H Qifang, F Lei, D Zhijun, et al., Application of modified analytic hierarchy process based on fuzzy comprehensive evaluation to landside risk assessment. J Yangtze River Scientific Res Institute **31**(05), 29–33 (2014) +38
13. T Yuejin, C Yingwu, Y Jinxian, *System Engineering Principle* (National University of Defense Technology press, Changsha), p. 2003
14. J Jing, Study of robustness in the world. J. Syst. Eng. **20**(02), 153–159 (2005)
15. F Zhiping, Y Tianhui, Z Quan, The sensitivity analysis to attribute values based on the additive weighting model in multiple attribute decision making. J Northeastern Univ (Natural Science). **23**(01), 83–86 (2002)
16. W Xianpeng, H Shengxiang, L Guanqing, Application of combined model based on particle swarm optimization in deformation analysis. Eng Surv Mapp **2**(01), 73–76 (2017)