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Bit error rate performance analysis of AC-MAP in multiple input single output wireless relay network

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Abstract

In this paper, we propose a joint decoding scheme called AC-MAP decoder for multiple input single output (MISO) wireless cooperative communication network that consists of single source, single relay, and single destination. The proposed scheme is based on both Alamouti combining (AC) scheme and maximum a posteriori (MAP) decoder and is used to estimate the data at the destination. The AC-MAP decoder is optimal in the sense that it minimizes the end-to-end bit error rate (BER). In order to analyze performance of the proposed decoder, we derive a closed form expression for the upper bound (UB) on the end-to-end error probability. Distances between system nodes, transmit energy, and channel noise and fading effects are considered in the derivation of the UB. Numerical results show that the closed form UB is very tight and it almost coincides with the exact BER results obtained from simulations. Therefore, we use the derived UB expression to study the effects of the relay position on the BER performance and to find the optimal location of the relay node.

Keywords: MISO relay network, MAP decoder, BER, Upper bound

1 Introduction

In recent years, cooperative communication is gaining a significant attention where relay nodes can collaborate with the users to enhance the wireless network performance. Cooperative relaying exploits the broadcast nature offered by the wireless medium where transmitted signals can be received, processed, and retransmitted by any node in the neighborhood of the source. The relay node could be a fixed node utilized by the network or another user that acts as a partner. In the second case, many partners may be available for each user to choose from which makes partner assignment important for better BER performance. Also, the energy allocation, for both user and relay in both cases, is important if the total energy is constrained. Cooperative communications provide a substantial improvement in the performance of the wireless networks in terms of rate (spectral efficiency or bandwidth) and reliability (diversity gain) [1–8]. This improvement can lead to the extension of the coverage and reduction in consumed energy. Cooperative relaying can have a great value in many systems such as ad hoc networks and next generation cellular and wireless local area networks. In cooperative diversity, relay nodes retransmit the signal received from the source which allows the receiver node to average channel variations resulting from fading and shadowing [1]. Several cooperation protocols have been proposed in the literature such as decode and forward (DF), amplify and forward (AF), and compress and forward (CF).

In this paper, we adopt the decode and forward (DF) cooperation protocol. In DF, the data transmission occurs over two phases. In the first phase, the source transmits its data to the intended destination. Because of the broadcast nature of the wireless medium, the signals from the source can be received by relay nodes. In the second phase, relay nodes decode the received signals and then forward the decoded data to the destination. Since the channel between the source and the relay is not necessarily error-free, a decoding error may occur at the relay node. Therefore, the data received at the destination from the relay node may not provide the expected information about the data of the source. Accordingly,

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the diversity gain expected from the overall network may not be achieved. Several techniques were proposed to enable the destination to estimate the data transmitted by the source. In those techniques, the destination first combines the received signals and then uses the combined signal to detect the data of the source. Several combining techniques were proposed such equal gain combining (EGC), maximum ratio combining (MRC), and selection combining (SC). These combining techniques would provide a BER performance similar to that provided by traditional diversity techniques (i.e., when the destination receives multiple copies of the same signal directly from the source) if the link between the source and relay (S-R link) is error-free; otherwise, it will lead to an error floor [9]. Error-free link can be achieved using automatic repeat request (ARQ) protocol at the relay node [10]. However, this will increase the overhead and, accordingly, reduces the network throughput. Several schemes have been proposed in the literature to decode the data at the destination taking into consideration the error in the S-R link. The error performance of the cooperative communications with decode-and-forward protocol with different combining techniques was also investigated in the literature. The end-to-end performance of wireless communication systems with relays over Rayleigh fading channels was studied in [11] when the direct link between the source and the destination does not exist. A general framework for ML detection of both coherent and noncoherent uncoded cooperative diversity was presented in [12] where the authors derived a high SNR approximations based on the closed-form BER expressions. An exact error analysis for decode and forward cooperation with maximal ratio combining in Nakagami fading was provided in [13]. The authors of [14] derived an analytical expression for the symbol error probability of the DF protocol with selection combining for M-ary phase-shift keying in Rayleigh fading environment.

The maximum a posteriori (MAP) decoder was proposed and its BER performance was analyzed in [15, 16] for single input single output (SISO) wireless relay network. Unlike other decoders, the receiver does not use any combining techniques. However, it considers the data received from the source together with the data received from the relay node as a codeword. The decoding rule is to find the codeword that maximizes the a posterior probability. Since the error probability in the source-to-relay link is not necessarily 0.5, the codewords received at the destination are not equiprobable. Therefore, the MAP decoding rule can not be simplified to the maximum likelihood (ML) decoding rule. Hence, the error probability of the S-R link is considered in the decoding process and, accordingly, the MAP decoder provides optimal performance in terms of BER. In this paper, we first modify the MAP decoder to support MISO wireless relay network. We find that the straightforward modification of the MAP technique will increase the decoding complexity at the destination. Therefore, we propose a joint decoding scheme called AC-MAP decoder that is based on both Alamouti combining (AC) scheme and maximum a posterior (MAP) decoder to estimate the data at the destination. The proposed scheme mitigates the complexity problem of the MAP decoder. The proposed scheme is optimal in the sense that it minimizes the end-to-end BER. We also derive a closed from expression for the upper bound on the bit error probability of the decode-and-forward cooperation protocol with the proposed AC-MAP decoder. The derived upper bound takes into account the SNR of all links (i.e., source-to-relay, source-to-destination, and relay-to-destination).

One of the end goals of this paper is to find the optimal position of the relay node. The problem of relay positioning and partner assignment was proposed in wireless cooperative and sensor networks to improve the overall system performance and energy efficiency. Optimal positioning of relay node is a very challenging and complex problem [17]. In order to address the complexity problem, two approaches were proposed in the literature. The first approach is to provide suboptimal solutions supported by heuristics [18]. The second approach is to find the optimal position by considering specific performance metrics [19, 20]. In this paper, we adopt the second approach to find the optimal position of the relay node in the case of fixed relay and to find the best partner if the source has many partners to choose from. The performance metric we use in this analysis is the bit error rate. Since the proposed AC-MAP decoding scheme is optimal and the derived upper bound is very tight, the closed form expression for the UB on the BER is used to find the optimal location the relay node.

The remainder part of this paper is organized as follows. The system model is described in Section 2. The proposed decoding scheme is described in Section 3. The BER performance analysis for the AC-MAP decoder is presented in Section 4. Positioning of relay node is presented in Section 5. Section 6 presents numerical results and discussions. Finally, the conclusions are drawn in Section 7.

2 System model

We consider a relay network composed of a source (S) equipped with two antennas, a relay (R), and a destination (D) as shown in Fig. 1. The data transmission occurs over two phases. In the first phase, the source sends its data to the destination using the Alamouti code [21] to achieve transmit diversity. Using this scheme, two symbols s_0 and s_1 are transmitted simultaneously twice in two time slots t_0 and t_1. In time slot t_0, the two antennas of the source A_1 and A_2 transmit signals corresponding to s_0 and s_1,

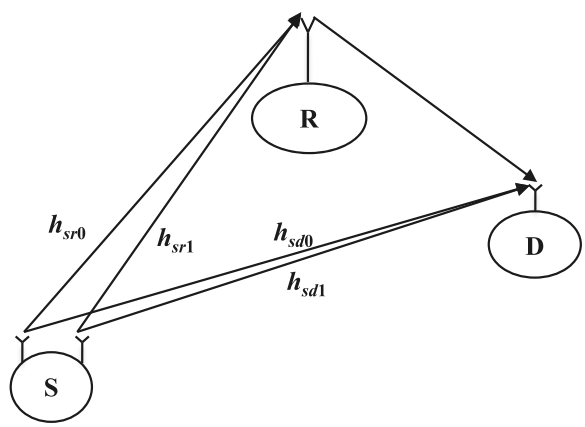

Fig. 1 System model

respectively. In the next time slot t_1, the two antennas A_1 and A_2 transmit signals corresponding to $-s_1{}^*$ and $s_0{}^*$, respectively, where "$*$" denotes the complex conjugate. Due to the broadcast nature of the wireless medium, the relay also receives the data from the source (possibly with some errors). We assume that all data are sent using BPSK modulation scheme and the source generates its bits with equal probability, i.e., $p(bs = 0) = p(bs = 1) = 0.5$ where b_s is the source bit. We assume that all channels are Rayleigh flat fading with additive white Gaussian noise (AWGN). The channel gain is assumed to be constant over two consecutive time slots which is necessary for the decoding of the Alamouti-transmitted signals. The signals received at the end of the first phase (two time slots) by the relay and the destination, respectively, are

$$y_{sr0} = h_{sr0}\sqrt{d_{sr}^{-m}}s_0 + h_{sr1}\sqrt{d_{sr}^{-m}}s_1 + n_{r0}, \tag{1}$$

$$y_{sr1} = -h_{sr0}\sqrt{d_{sr}^{-m}}s_1{}^* + h_{sr1}\sqrt{d_{sr}^{-m}}s_0{}^* + n_{r1}, \tag{2}$$

$$y_{sd0} = h_{sd0}\sqrt{d_{sd}^{-m}}s_0 + h_{sd1}\sqrt{d_{sd}^{-m}}s_1 + n_{sd0}, \tag{3}$$

$$y_{sd1} = -h_{sd0}\sqrt{d_{sd}^{-m}}s_1{}^* + h_{sd1}\sqrt{d_{sd}^{-m}}s_0{}^* + n_{sd1} \tag{4}$$

where

- y_{sr0} is the signal received at the relay node in the first phase in time slot t_0.
- y_{sr1} is the signal received at the relay node in the first phase in time slot t_1.
- y_{sd0} is the signal received at the destination in the first phase in time slot t_0.
- y_{sd1} is the signal received at the destination in the first phase in time slot t_1.
- $s_i \in \{+\sqrt{E_s}, -\sqrt{E_s}\}$ is the BPSK modulated signal of the ith symbol sent by the source, $i \in \{0, 1\}$, where E_s is the transmit energy per the source bit.
- h_{srj} and $h_{sdj}, j \in \{0, 1\}$, are the channel fading gains of the S-R and S-D links, respectively.
- d_{sr} and d_{sd} are the lengths of the S-R and S-D links, respectively.
- m is the path loss exponent.
- $n_{ri}, i \in \{0, 1\}$ is the AWGN noise at the relay node which has zero mean and variance $N_0/2$.
- n_{sdi} is the AWGN noise at the destination node which has zero mean and variance $N_0/2$.

In the second phase, the relay node decodes the data received from the source using Alamouti combining scheme by first calculating the decision variables r_0 and r_1 as follows

$$\begin{aligned} r_0 =& h_{sr0}^* y_{sr0} + h_{sr1} y_{sr1}^* \\ =& (h_{sr0}^2 + h_{sr1}^2)s_0 + h_{sr0}^* n_{r0} + h_{sr1} n_{r1}^*, \end{aligned} \tag{5}$$

$$\begin{aligned} r_1 =& h_{sr1}^* y_{sr0} - h_{sr0} y_{sr1}^* \\ =& (h_{sr0}^2 + h_{sr1}^2)s_1 - h_{sr0} n_{r1}^* + h_{sr1}^* n_{r0} \end{aligned} \tag{6}$$

and then estimates the transmitted bits as follows

$$br_i = \begin{cases} 0 & r_i \geq 0 \\ 1 & r_i < 0 \end{cases} \tag{7}$$

where br_i represents an estimate of the source bit bs_i at the relay node. Then, the relay forwards the decoded data to the destination in two successive time slots. The signals received at the destination from the relay are

$$y_{rd0} = h_{rd0}\sqrt{d_{rd}^{-m}}\hat{s}_0 + n_{rd0}, \tag{8}$$

$$y_{rd1} = h_{rd1}\sqrt{d_{rd}^{-m}}\hat{s}_1 + n_{rd1} \tag{9}$$

where

- y_{rd0} is the signal received at destination from the relay node in the first time slot of the second phase (third time slot in total).
- y_{rd1} is the signal received at destination from the relay node in the second time slot of the second phase (fourth time slot in total).
- $\hat{s}_i \in \{+\sqrt{E_r}, -\sqrt{E_r}\}$ is the BPSK modulated signal of the ith symbol estimated and sent by the relay, $i \in \{0, 1\}$, where E_r is the transmit energy per the relay bit.
- h_{rdi} is the channel gain of the R-D in the i-th time slot of the second phase, where $i \in \{0, 1\}$.
- d_{rd} is the length of the R-D link.
- n_{rdi} is the AWGN noise at the destination node which has zero mean and variance $N_0/2$.

Let the random vector $e = [e_0\ e_1]^T$ where $e_i \in \{0, 1\}$ captures the error events on the source-to-relay channels, i.e., $b_{r_i} = b_{s_i} \oplus e_i$, where $\oplus$ denotes the binary XOR (exclusive OR) operation. Hence, $e_i = 1$ means $br_i \neq bs_i$, i.e., the

source bit is received in error at the relay node. In the case of BPSK modulation over flat fading channel with AWGN, the probability of error per bit in the source-relay link, i.e., $P(e_i = 1)$, is given by [22]

$$P_{e_{sr}} = 0.25 - \sqrt{\frac{\gamma_{sr}}{1.5 + \gamma_{sr}}} + 0.75\sqrt{\frac{\gamma_{sr}}{2 + \gamma_{sr}}} \tag{10}$$

where $\gamma_{sr} = d_{sr}^{-m} E_s/N_0$ is the average receive SNR of the link between the source and the relay.

The received signals at the destination can be written in the matrix form as follows

$$\mathbf{y} = H\mathbf{s} + \mathbf{n} \tag{11}$$

where the received vector $\mathbf{y} = [y_{sd0}\ y_{sd1}^*\ y_{rd0}\ y_{rd1}]^T$, the transmitted vector $\mathbf{s} = [s_0\ s_1\ \hat{s}_0\ \hat{s}_1]^T$, the noise vector $\mathbf{n} = [n_{sd0}\ n_{sd1}^*\ n_{rd0}\ n_{rd1}]^T$, and the channel matrix H is given by

$$H = \begin{bmatrix} h_{sd0}\sqrt{d_{sd}^{-m}} & h_{sd1}\sqrt{d_{sd}^{-m}} & 0 & 0 \\ h_{sd1}^*\sqrt{d_{sd}^{-m}} & -h_{sd0}^*\sqrt{d_{sd}^{-m}} & 0 & 0 \\ 0 & 0 & h_{rd0}\sqrt{d_{rd}^{-m}} & 0 \\ 0 & 0 & 0 & h_{rd1}\sqrt{d_{rd}^{-m}} \end{bmatrix} \tag{12}$$

3 Decoding schemes used at the destination

In the considered MISO cooperative communication network, the source node is equipped with two antennas while the relay and the destination nodes are equipped with single antenna. To achieve diversity gain, the source transmits its data using Alamouti space time block code (STBC). The relay node uses the decode and forward (DF) cooperation protocol in order to increase the reliability of the source data at the destination. Hence, the destination receives four signals (two from the source and two form the relay node) in four time slots. The destination uses these four signals jointly to decode the data sent from the source (i.e., b_{s_0} and b_{s_1}). In this section, firstly, we present the maximum a posterior (MAP) decoding scheme for MISO relay network. Secondly, we propose a new decoding scheme that mitigates the complexity problem of the MAP decoder.

3.1 Maximum a posterior decoder

In this section, we present the MAP decoding scheme used by the destination to estimate the data sent from the source for the MISO relay network under consideration. The MAP decoding scheme is optimal in the sense that it minimizes the error probability at the destination. Let $\mathbf{b} = [bs_0\ bs_1\ br_0\ br_1]^T$ represents the bits vector (codeword) of the data transmitted from the source and relay nodes. If the S-R link is error-free, then $br_0 = bs_0$ and $br_1 = bs_1$ and, accordingly, there will be only four possible values of this vector. Since the S-R link is not error-free, there are sixteen possible values for this vector ranging from $\mathbf{b}_1 = [0\,0\,0\,0]^T$ to $\mathbf{b}_{16} = [1\,1\,1\,1]^T$ based on the values of b_{s_i} and e_i. Since, in general, $P(e_i = 1) \neq P(e_i = 0)$, these 16 vectors (codewords) are not equiprobable. For example, the probability of transmitting the vector $\mathbf{b}_5 = [0\,1\,0\,0]^T$ is given by

$$\begin{aligned} P(\mathbf{b}_5) &= P(bs_0 = 0, bs_1 = 1, br_0 = 0, br_1 = 0) \\ &= P(br_0 = 0, br_1 = 0 | bs_0 = 0, bs_1 = 1) \\ &\times P(bs_0 = 0, bs_1 = 1) \end{aligned} \tag{13}$$

Since the source bits are independent and equiprobable, then

$$P(bs_0 = 0, bs_1 = 1) = \frac{1}{4} \tag{14}$$

The first part of (13) is given by

$$\begin{aligned} &P(br_0 = 0, br_1 = 0 | bs_0 = 0, bs_1 = 1) \\ &= P(b_{s_0} \oplus e_0 = 0, b_{s_1} \oplus e_1 = 0 | bs_0 = 0, bs_1 = 1) \\ &= P(e_0 = 0, e_1 = 1) \\ &= P_{e_{sr}}(1 - P_{e_{sr}}) \end{aligned} \tag{15}$$

Substituting from (15) and (14) into (13) yields

$$P(\mathbf{b}_5) = \frac{P_{e_{sr}}(1 - P_{e_{sr}})}{4} \tag{16}$$

Similarly, the probability of transmitting a specific vector (codeword) $\mathbf{b}_i$ is given by

$$P(\mathbf{b}_i) = \begin{cases} \frac{(1-P_{e_{sr}})^2}{4} & i = 1, 6, 11, 16 \\ \frac{P_{e_{sr}}^2}{4} & i = 4, 7, 10, 13 \\ \frac{P_{e_{sr}}(1-P_{e_{sr}})}{4} & i = \text{otherwise} \end{cases} \tag{17}$$

The MAP decoder finds the transmitted vector $\mathbf{b}_i$ that maximizes the a posterior probability $P(\mathbf{b}_i|\mathbf{y})$. An estimate of the transmitted vector is given by

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}_i} P(\mathbf{b}_i|\mathbf{y}) \tag{18}$$

and the maximization process is performed over the 16 possible vectors (codewords). Applying Bayes theorem to (18) yields

$$\begin{aligned} \hat{\mathbf{b}} &= \arg\max_{\mathbf{b}_i} \frac{P(\mathbf{y}|\mathbf{b}_i)P(\mathbf{b}_i)}{P(\mathbf{y})} \\ &= \arg\max_{\mathbf{b}_i} P(\mathbf{y}|\mathbf{b}_i)P(\mathbf{b}_i) \end{aligned} \tag{19}$$

where [23]

$$P(\mathbf{y}|\mathbf{b}_i) = \frac{1}{(\pi N_0)} e^{-||\mathbf{y} - H\mathbf{s}_i||^2/N_0} \tag{20}$$

and $\mathbf{s}_i$ is the modulated vector that corresponds to the vector $\mathbf{b}_i$, e.g., $\mathbf{s}_3 = [-\sqrt{E_s}\ -\sqrt{E_s}\ \sqrt{E_r}\ -\sqrt{E_r}]^T$. Substituting from (20) into (19) yields

$$\begin{aligned}\hat{\mathbf{b}} =& \arg\max_{\mathbf{b}_i} \frac{1}{(\pi N_0)} e^{-||\mathbf{y}-H\mathbf{s}_i||^2/N_0} P(\mathbf{b}_i) \\ =& \arg\min_{\mathbf{b}_i} \left(||\mathbf{y}-H\mathbf{s}_i||^2 - N_0 \log(P(\mathbf{b}_i))\right)\end{aligned} \tag{21}$$

For the described system model, in order for the destination to decode the two bits sent from the source, it has to search a space composed of sixteen vectors each of length four bits. Although the MAP decoder is optimal in the sense of minimizing the error rate, its complexity is high especially when the number antennas, network nodes, and/or the modulation order increases. In the following section, we present a less complex joint decoding scheme that combines both MAP and Alamouti decoding scheme at the destination to retrieve the source bits.

3.2 Proposed AC-MAP decoder

In this section, we present the joint decoding scheme (AC-MAP decoder) that is based on both Alamouti combining scheme and MAP decoder. The AC-MAP decoder mitigates the complexity problem through utilization of the Alamouti combining scheme in reducing the size of the search space used by the MAP decoder while maintaining the same optimal BER performance. The AC-MAP scheme is a two-stage decoder. In the first stage, Alamouti combining scheme is used on y_{sd0} and y_{sd1} given by (3) and (4) to get two signals corresponding to s_0 and s_1 as follows

$$\begin{aligned}d_0 =& h^*_{sd0} y_{sd0} + h_{sd1} y^*_{sd1} \\ =& (h^2_{sd0} + h^2_{sd1}) s_0 + h^*_{sd0} n_{sd0} + h_{sd1} n^*_{sd1},\end{aligned} \tag{22}$$

$$\begin{aligned}d_1 =& h^*_{sd1} y_{sd0} - h_{sd0} y^*_{sd1} \\ =& (h^2_{sd0} + h^2_{sd1}) s_1 - h_{sd0} n^*_{sd1} + h^*_{sd1} n_{sd0}\end{aligned} \tag{23}$$

In the second stage, MAP decoder is used to retrieve the source symbols separately where d_0 and y_{rd0} are used to decode s_0 and d_1 and y_{rd1} are used to decode s_1. The signals used for decoding s_0 can be written in the matrix form as follows

$$\mathbf{y} = H\mathbf{s} + \mathbf{w} \tag{24}$$

where $\mathbf{y} = [d_0 \; y_{rd0}]^T$, $\mathbf{s} = [s_0 \; \hat{s}_0]^T$, equivalent noise vector $\mathbf{w} = [h^*_{sd0} n_{sd0} + h_{sd1} n^*_{sd1} \; n_{rd0}]^T$, and the equivalent channel matrix H is given by

$$H = \begin{bmatrix} (h^2_{sd0} + h^2_{sd1})\sqrt{d^{-m}_{sd}} & 0 \\ 0 & h_{rd0}\sqrt{d^{-m}_{rd}} \end{bmatrix} \tag{25}$$

Let $\mathbf{b} = [bs_0 \; br_0]^T$ represents the transmitted bits vector corresponding to s_0. Since the S-R link is not error-free, there are four possible values for this vector $\mathbf{b}_1 = [0 \; 0]^T$, $\mathbf{b}_2 = [0 \; 1]^T$, $\mathbf{b}_3 = [1 \; 0]^T$, and $\mathbf{b}_4 = [1 \; 1]^T$ based on the values b_{s0} and e_0. Since, in general, $P(e_0 = 1) \neq P(e_0 = 0)$, these four vectors are not equiprobable. The probability of transmitting a specific vector $\mathbf{b}_i$ is given by

$$P(\mathbf{b}_i) = \begin{cases} \frac{1-P_{esr}}{2} & i = 1, 4 \\ \frac{P_{esr}}{2} & i = 2, 3 \end{cases} \tag{26}$$

The MAP decoder finds the transmitted vector $\mathbf{b}_i$ that maximizes the a posterior probability $P(\mathbf{b}_i|\mathbf{y})$. An estimate of the transmitted vector is given by

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}_i} P(\mathbf{b}_i|\mathbf{y}) \tag{27}$$

Applying Bayes theorem on (27) yields

$$\begin{aligned}\hat{\mathbf{b}} =& \arg\max_{\mathbf{b}_i} \frac{P(\mathbf{y}|\mathbf{b}_i)P(\mathbf{b}_i)}{P(\mathbf{y})} \\ =& \arg\max_{\mathbf{b}_i} P(\mathbf{y}|\mathbf{b}_i)P(\mathbf{b}_i)\end{aligned} \tag{28}$$

where $P(\mathbf{y}|\mathbf{b}_i)$ has a multivariate normal distribution given by

$$P(\mathbf{y}|\mathbf{b}_i) = \frac{1}{\left(2\pi\sqrt{|\Sigma|}\right)} exp\left(-0.5(\mathbf{y} - H\mathbf{s}_i)^T \Sigma^{-1} (\mathbf{y} - H\mathbf{s}_i)\right), \tag{29}$$

the 2×2 covariance matrix Σ is given by

$$\Sigma = \begin{bmatrix} \frac{N_0}{2}(h^2_{sd0} + h^2_{sd1}) & 0 \\ 0 & \frac{N_0}{2} \end{bmatrix}, \tag{30}$$

and $\mathbf{s}_i$ is the modulated vector corresponding to the vector $\mathbf{b}_i$, e.g., $\mathbf{x}_3 = [\sqrt{E_s} \; -\sqrt{E_r}]^T$. In order to simplify the calculations of (28) and the analysis of the bit error rate, we redefine the received vector $\mathbf{y}$ as follows

$$\mathbf{y} = [\frac{d_0}{\sqrt{h^2_{sd0} + h^2_{sd1}}} \; y_{rd0}]^T \tag{31}$$

Hence, the covariance matrix of the noise vector would be $\Sigma = (N_0/2)\iota_2$, where ι_2 is the 2×2 identity matrix, and the equivalent channel matrix H would be given by

$$H = \begin{bmatrix} \sqrt{(h^2_{sd0} + h^2_{sd1}) d^{-m}_{sd}} & 0 \\ 0 & h_{rd0}\sqrt{d^{-m}_{rd}} \end{bmatrix} \tag{32}$$

Therefore, (28) can be rewritten as follows

$$\begin{aligned}\hat{\mathbf{b}} =& \arg\max_{\mathbf{b}_i} \frac{1}{(\pi N_0)} e^{-||\mathbf{y}-H\mathbf{s}_i||^2/N_0} P(\mathbf{b}_i) \\ =& \arg\min_{\mathbf{b}_i} \left(||\mathbf{y} - H\mathbf{s}_i||^2 - N_0 \log(P(\mathbf{b}_i))\right)\end{aligned} \tag{33}$$

Similarly, the signals d_1 and y_{rd1} can be used to decode s_1. Using two signals to decode each symbol reduces the size of the search space to four vectors each of length of two bits and, accordingly, reduces the decoding complexity. Since the first stage (Alamouti combining) preserves all the information about the data transmitted by the source, both MAP and AC-MAP decoding schemes would provide the same performance. Figure 2 shows simulation results for the BER against the source transmit SNR (E_s/N_0) when the relay transmit SNR (E_r/N_0) is constant for both MAP and AC-MAP decoders. As expected, the

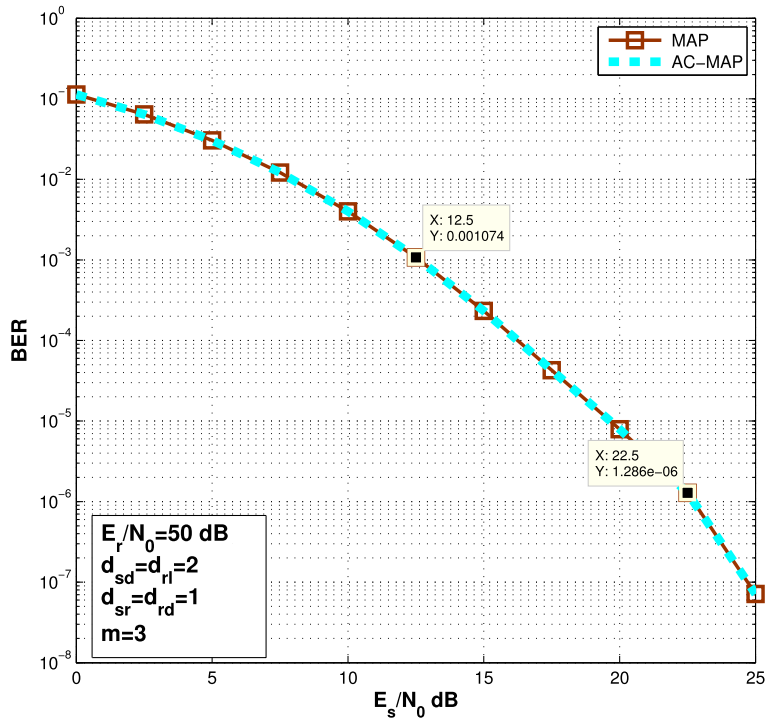

Fig. 2 MAP vs. AC-MAP BER performance

figure shows that the both MAP and AC-MAP decoding schemes provide exactly the same BER performance.

The proposed AC-MAP decoding scheme can be easily modified to support higher order modulation. However, the complexity will dramatically increase because of the massive increase in the size of the search space. For example, in 16-QAM, the search space is composed of 256 vectors (codewords) instead of 4 in the case of BPSK.

4 BER analysis of the AC-MAP decoder

In this section, we derive the upper bound (UB) on the bit error probability of the AC-MAP decoding scheme.

4.1 Derivation of the upper bound

The destination uses (33) to estimate the transmitted codeword. Since the first bit of the codeword is an estimate of the bit coming directly from the source, the bit error probability would be given by

$$\begin{aligned} P_E \leq & (P(\mathbf{b}_1 \to \mathbf{b}_3) + P(\mathbf{b}_1 \to \mathbf{b}_4))\, P(\mathbf{b}_1) \\ & + (P(\mathbf{b}_2 \to \mathbf{b}_3) + P(\mathbf{b}_2 \to \mathbf{b}_4))\, P(\mathbf{b}_2) \\ & + (P(\mathbf{b}_3 \to \mathbf{b}_1) + P(\mathbf{b}_3 \to \mathbf{b}_2))\, P(\mathbf{b}_3) \\ & + (P(\mathbf{b}_4 \to \mathbf{b}_1) + P(\mathbf{b}_4 \to \mathbf{b}_2))\, P(\mathbf{b}_4) \end{aligned} \tag{34}$$

where $P(\mathbf{b}_k \to \mathbf{b}_l)$ is the pairwise error probability of confusing $\mathbf{b}_k$ with $\mathbf{b}_l$ when $\mathbf{b}_k$ is transmitted and when these are the only two hypothesis. When the vector $\mathbf{b}_k$ is transmitted, the received vector would be

$$\mathbf{y}_k = H\mathbf{s}_k + \mathbf{w} \tag{35}$$

and, according to (33), the probability $P(\mathbf{b}_k \to \mathbf{b}_l)$ would be given by

$$\begin{aligned} P(\mathbf{b}_k \to \mathbf{b}_l) =& P\Big(||\mathbf{y}_k - H\mathbf{s}_l||^2 - N_0 \log(P(\mathbf{b}_l)) \\ & < ||\mathbf{y}_k - H\mathbf{s}_k||^2 - N_0 \log(P(\mathbf{b}_k)) \Big) \\ =& P\Big(||H(\mathbf{x}_k - \mathbf{s}_l) + \mathbf{w}||^2 \\ & + N_0 \log \frac{P(\mathbf{b}_k)}{P(\mathbf{b}_l)} < ||\mathbf{w}||^2 \Big) \\ =& P\Big((< \mathbf{w}, H(\mathbf{s}_k - \mathbf{s}_l) >) \\ & > \frac{||H(\mathbf{s}_k - \mathbf{s}_l)||^2}{2} + \frac{N_0}{2} \log \frac{P(\mathbf{b}_k)}{P(\mathbf{b}_l)} \Big) \end{aligned} \tag{36}$$

When $\mathbf{h} = [h_{sd0}\ h_{sd1}\ h_{rd0}]^T$ is given, $< \mathbf{w}, H(\mathbf{s}_k - \mathbf{s}_l) >$ would be a Gaussian random variable with zero mean and variance $\frac{N_0}{2}||H(\mathbf{s}_k - \mathbf{s}_l)||^2$. Accordingly,

$$P(\mathbf{b}_k \to \mathbf{b}_l|\mathbf{h}) = Q\left(\frac{||H(\mathbf{s}_k - \mathbf{s}_l)||}{\sqrt{2N_0}} + \frac{\sqrt{N_0/2}\log(\alpha_{kl})}{||H(\mathbf{s}_k - \mathbf{s}_l)||} \right) \tag{37}$$

where $\alpha_{kl} = P(\mathbf{b}_k)/P(\mathbf{b}_l)$. From, (26), there will three possible values for α_{kl} as follows

$$\begin{aligned} \alpha_{14} =& \alpha_{41} = \alpha_{23} = \alpha_{32} = 1 \\ \alpha_{13} = \alpha_{42} =& \frac{1 - P_{e_{sr}}}{P_{e_{sr}}} \\ \alpha_{31} = \alpha_{24} =& \frac{P_{e_{sr}}}{1 - P_{e_{sr}}} \end{aligned} \tag{38}$$

From (37), we notice that the probability $P(\mathbf{b}_k \to \mathbf{b}_l|\mathbf{h})$ depends on:
1- The *Hamming distance* between $\mathbf{b}_k$ and $\mathbf{b}_l$, w_{kl}, (i.e., the number of positions at which $\mathbf{b}_k$ and $\mathbf{b}_l$ are different).
2- Whether $\mathbf{b}_k$ and $\mathbf{b}_l$ have the same probability.

The four possible codewords are $\mathbf{b}_1 = [0\ 0]^T$, $\mathbf{b}_2 = [0\ 1]^T$, $\mathbf{b}_3 = [1\ 0]^T$, and $\mathbf{b}_4 = [1\ 1]^T$. From (26), we find that codewords with equal probability (i.e., $\alpha_{kl} = 1$) have a hamming distance $w_{kl} = 2$ (e.g., codewords $\mathbf{b}_1$ and $\mathbf{b}_4$). Accordingly, and from (37) and (38), it is straightforward to show that $P(\mathbf{b}_k \to \mathbf{b}_l)$ does not depend on α_{kl} when $w_{kl} = 2$. Let $P(\mathbf{b}_k \to \mathbf{b}_l) = P_1(\alpha_{kl})$ when the hamming distance $w_{kl} = 1$ and $P(\mathbf{b}_k \to \mathbf{b}_l) = P_2$ when $w_{kl} = 2$. Hence, substituting from (26) into (34) yields

$$\begin{aligned} P_E \le& (P_1(\alpha_{13}) + P_2)\frac{1 - P_{e_{sr}}}{2} \\ &+ (P_2 + P_1(\alpha_{24}))\frac{P_{e_{sr}}}{2} \\ &+ (P_1(\alpha_{31}) + P_2)\frac{P_{e_{sr}}}{2} \\ &+ (P_2 + P_1(\alpha_{42}))\frac{1 - P_{e_{sr}}}{2} \end{aligned} \tag{39}$$

which can be rewritten as follows

$$\begin{aligned} P_E \le& P_2 + (P_1(\alpha_{31}) + P_1(\alpha_{24}))\frac{P_{e_{sr}}}{2} \\ &+ (P_1(\alpha_{13}) + P_1(\alpha_{42}))\frac{1 - P_{e_{sr}}}{2} \end{aligned} \tag{40}$$

Since $P_1(\alpha_{31}) = P_1(\alpha_{24})$ and $P_1(\alpha_{13}) = P_1(\alpha_{42})$, the upper bound on the error probability that is equal to right-hand side of (40) would be

$$P_{UB} = P_2 + P_1(\alpha_{31})P_{e_{sr}} + P_1(\alpha_{13})(1 - P_{e_{sr}}) \tag{41}$$

4.2 Derivation P_2

In order to derive P_2, Eq. (37) is considered for $\mathbf{b}_1 = [0\ 0]^T$, $\mathbf{b}_4 = [1\ 1]^T$ as follows:

$$P(\mathbf{b}_1 \to \mathbf{b}_4|\mathbf{h}) = Q\left(\sqrt{2(h_{sd0}^2 + h_{sd1}^2)d_{sd}^{-m}\frac{E_s}{N_0} + 2h_{rd}^2 d_{rd}^{-m}\frac{E_r}{N_0}}\right) \tag{42}$$

Let $\gamma_{sd} = d_{sd}^{-m}\frac{E_s}{N_0}$ and $\gamma_{rd} = d_{rd}^{-m}\frac{E_r}{N_0}$, then

$$\begin{aligned} P(\mathbf{b}_1 \to \mathbf{b}_4|\mathbf{h}) &= Q\left(\sqrt{2h_{rd}^2\gamma_{rd} + 2(h_{sd0}^2 + h_{sd1}^2)\gamma_{sd}}\right) \\ &= Q\left(\sqrt{2x}\right) \end{aligned} \tag{43}$$

where $x = h_{rd}^2\gamma_{rd} + (h_{sd0}^2 + h_{sd1}^2)\gamma_{sd}$. In order to find P_2, we average (43) over the distribution of x. Since all channels have the same distribution, $\{\forall k, l : w_{kl} = 2\}$, we have

$$\begin{aligned} P_2 =& \int_0^\infty Q\left(\sqrt{2x}\right) f_X(x)dx \\ =& 0.5\int_0^\infty erfc\left(\sqrt{x}\right) f_X(x)dx \end{aligned} \tag{44}$$

The distribution of x is given by [24]

$$\begin{aligned} f_X(x) =& \frac{\gamma_{rd}}{(\gamma_{rd} - \gamma_{sd})^2}\left(e^{-x/\gamma_{rd}} - e^{-x/\gamma_{sd}}\right) \\ &- \frac{1}{\gamma_{sd}(\gamma_{rd} - \gamma_{sd})}xe^{-x/\gamma_{sd}} \end{aligned} \tag{45}$$

Substituting from (45) into (44) for $f_X(x)$ yields

$$\begin{aligned} P_2 =& \frac{\gamma_{rd}}{2(\gamma_{rd} - \gamma_{sd})^2}\int_0^\infty erfc\left(\sqrt{x}\right)\left(e^{-x/\gamma_{rd}} - e^{-x/\gamma_{sd}}\right)dx \\ &- \frac{1}{2\gamma_{sd}(\gamma_{rd} - \gamma_{sd})}\int_0^\infty erfc\left(\sqrt{x}\right)xe^{-x/\gamma_{sd}}dx \end{aligned} \tag{46}$$

In the Appendix, we derive closed form expressions for the integrals of (46). Applying these derivation results to (46) yields

$$\begin{aligned} P_2 =& \frac{\gamma_{rd}}{2(\gamma_{rd} - \gamma_{sd})^2}I_{11}(\gamma_{rd}, 0) \\ &- \frac{\gamma_{rd}}{2(\gamma_{rd} - \gamma_{sd})^2}I_{11}(\gamma_{sd}, 0) \\ &- \frac{1}{2\gamma_{sd}(\gamma_{rd} - \gamma_{sd})}I_{21}(\gamma_{sd}, 0) \end{aligned} \tag{47}$$

where

$$I_{11}(\gamma_{rd}, 0) = \frac{1 - \Gamma_{rd}}{2}, \tag{48}$$

$$I_{11}(\gamma_{sd}, 0) = \frac{1 - \Gamma_{sd}}{2}, \tag{49}$$

$$I_{21}(\gamma_{sd}, 0) = \frac{\gamma_{sd}(1 - \Gamma_{sd})}{2} - \frac{\Gamma_{sd}^3}{2} \tag{50}$$

where $\Gamma_{rd} = \sqrt{\gamma_{rd}/(1 + \gamma_{rd})}$ and $\Gamma_{sd} = \sqrt{\gamma_{sd}/(1 + \gamma_{sd})}$

4.3 Derivation $P_1(\alpha_{kl})$

In order to derive $P_1(\alpha_{kl})$, Eq. (37) is first considered for $\mathbf{b}_1 = [0\ 0]^T$ and $\mathbf{b}_3 = [1\ 0]^T$ and then we perform generalization as follows

$$P(\mathbf{b}_1 \to \mathbf{b}_3|\mathbf{h}) = Q\left(\sqrt{2({h_{sd1}}^2 + h_{sd2}^2)\gamma_{sd}} + \frac{log(\alpha_{13})}{2\sqrt{2({h_{sd1}}^2 + h_{sd2}^2)\gamma_{sd}}}\right) \quad (51)$$

Hence, $P_1(\alpha_{13})$ can be derived by averaging (51) over the distribution of $x = ({h_{sd1}}^2 + h_{sd2}^2)\gamma_{sd}$. Since all channels have the same distribution, $\{\forall k, l : w_{kl} = 1\}$, we have

$$P_1(\alpha_{kl}) = \int_0^\infty Q\left(\sqrt{2x} + \frac{\log(\alpha_{kl})}{2\sqrt{2x}}\right) f_X(x)dx = 0.5\int_0^\infty erfc\left(\sqrt{x} + \frac{\log(\alpha_{kl})}{4\sqrt{x}}\right) f_X(x)dx \quad (52)$$

where distribution of x is given by [24]

$$f_X(x) = \frac{1}{\gamma_{sd}^2} x e^{-x/\gamma_{sd}} \quad (53)$$

Substituting from (53) into (52) yields

$$P_1(\alpha_{kl}) = \frac{1}{2{\gamma_{sd}}^2}\int_0^\infty \text{erfc}\left(\sqrt{x} + \frac{b_{kl}}{\sqrt{x}}\right) x e^{-x/\gamma_{sd}} dx \quad (54)$$

where $b_{kl} = \log(\alpha_{kl})/4$. When the error probability of the S-R link is less than fifty percent (i.e., $P_{esr} < 0.5$), which is a reasonable assumption, and according to (26), we have

$$\begin{cases} b_{kl} > 0 & \text{if } k < l \\ b_{kl} < 0 & \text{if} k > l \end{cases} \quad (55)$$

Accordingly, and after applying the derivation results provided in the Appendix to (54), we would have

$$P_1(\alpha_{kl}) = \begin{cases} I_{21}(\gamma_{sd}, b_{kl}) & k < l \\ I_{22}(\gamma_{sd}, b_{kl}) & k > l \end{cases} \quad (56)$$

where

$$I_{21}(\gamma_{sd}, b_{kl}) = \left(\frac{\gamma_{sd}}{2}(1 - \Gamma_{sd}) - \Gamma_{sd}^2\left(\frac{\Gamma_{sd}}{2} - \frac{b_{kl}(1 - \Gamma_{sd})}{\Gamma_{sd}}\right)\right) \times e^{-2b_{kl}(1+\Gamma_{sd})/\Gamma_{sd}} \quad (57)$$

and

$$I_{22}(\gamma_{sd}, b_{kl}) = \gamma_{sd} - e^{2b_{kl}(1-\Gamma_{sd})/\Gamma_{sd}} \times \left(\frac{\gamma_{sd}}{2}(1 + \Gamma_{sd}) + \Gamma_{sd}^2\left(\frac{\Gamma_{sd}}{2} - \frac{b_{kl}(1 - \Gamma_{sd})}{\Gamma_{sd}}\right)\right) \quad (58)$$

5 Assignment and positioning of relay node

As will be discussed in Section 6, the derived upper bound on the error probability is tight and, accordingly, can be used to solve the relay assignment and positioning problems. In the problem of partner assignment, when a user (source) has many possible partners (relays) to choose from, the upper bound is calculated for each one and the relay node that achieves the minimum UB is selected as a partner. In the case of a fixed relay employed by the system, it is clear from (41), (47), and (56) that the error probability depends on average SNR's γ_{sr} and γ_{rd} of S-R and R-D links and, accordingly, on the lengths of these links, i.e., d_{sr} and d_{rd}. When the relay node is positioned close to the destination (far from the source), the good quality of the R-D link reduces the end-to-end error probability; however, the poor quality of the S-R link increases the error probability at the rely node and, accordingly, the end-to-end error probability. When the relay node is positioned close to the source (far from the destination), the poor quality of the R-D link increases the end-to-end error probability; however, the good quality of the S-R link reduces the error probability at the rely node and, accordingly, the end-to-end error probability. Therefore, there should be an optimal position of the relay node that minimizes the end-to-end error probability.

Referring to the derivations of Section 4, (41) can be rewritten as functions of source transmit SNR E_s/N_0, relay transmit SNR E_r/N_0, length S-R link d_{sr}, length S-D link d_{sd}, length R-D link d_{rd}, and the path loss exponent m as follows:

$$P_{UB} = \mathscr{F}\left(\frac{E_s}{N_0}, \frac{E_r}{N_0}, d_{sr}, d_{sd}, d_{rd}, m\right) \quad (59)$$

Although the relay node can be positioned at any point in the geographical area surrounding the destination and the source, it is straightforward to show that the error probability is minimized if the relay is positioned on the straight line connecting the source and the destination and, hence, $d_{sd} = d_{sr} + d_{rd}$. Accordingly, (59) can be written as follows

$$P_{UB} = \mathscr{F}\left(\frac{E_s}{N_0}, \frac{E_r}{N_0}, d_{sr}, d_{sd}, m\right) \quad (60)$$

Accordingly, parameters of the right hand side of (60) can be optimized to minimize the BER. In this paper, we are interested in studying the effect of relay node position on the error probability and in finding the optimal position that minimizes that probability when all other parameters are given. Therefore, (60) can be written as follows

$$P_{UB} = \mathscr{F}(d_{sr}) \quad (61)$$

Accordingly, the positioning problem turns out to be an optimization problem where d_{sr} is to be estimated for minimum P_{UB}. The problem can be modeled as a search problem in one dimensional search space and numerical solution can be used to find d_{sr} that minimizes the upper bound.

6 Numerical result and discussions

In this section, we present numerical results for both analysis and simulations. Although the AC-MAP is a two-stage decoder, the simulation results show that the decoding time of the AC-MAP takes only 18.4% of the traditional MAP decoding time. In all results, we assume that the lengths of the S-D link d_{sd} and the S-R-D link d_{rl} are equal (i.e., $d_{sd} = d_{sr} + d_{rd}$) and the path loss exponent is $m = 3.5$.

Figure 3 compares the bit error probability obtained from simulations with the upper bound given by (41). The BER is plotted against the length of S-R link d_{sr} (where $0 \leq d_{sr} \leq d_{sd}$) at different values of source and relay transmit SNR's E_s/N_0 and E_r/N_0, respectively. The source transmits from two antennas each with energy $E_s/2$. Three different cases are considered when $E_s > E_r$, $E_s < E_r$, and $E_s = E_r$. We find that the derived upper bound is very tight and almost coincides with the exact error probability obtained from simulations. We also find that, in each case, there is an optimal value of d_{sr} at which the BER is minimized. The optimal value of d_{sr} depends on the values of E_s and E_r.

Figure 3 also shows other two important results. The first, under a total transmit power constraint, it is better to allocate more power to the source than the relay while positioning the relay closer to the destination. The reason is that more source power improves the reliability of both the S-D and S-R links while the reliability of the R-D link is improved by making the relay closer to the destination. The second is related to the case when $E_s = E_r$ which shows that the optimum relay position is not at the middle of the S-R-D link; however, it is closer to the destination. The reason is that the source is equipped with two antennas which makes the quality of S-R link better than that of the R-D link.

Figure 4 shows the minimum BER at the optimal d_{sr} against the source transmit SNR (E_s/N_0) at different values of the relay transmit SNR (E_r/N_0). The figure shows that although the S-R link is not error free the AC-MAP decoder is capable of achieving the expected diversity order of three. The figure also shows that increasing the relay transmit power enhances the BER performance dramatically at lower values of E_s/N_0 while provides a little improvement at higher values of E_s/N_0. That is because when E_s/N_0 is small, the relay is located close to the source (far from the destination) and it receives the data with high reliability, hence, retransmitting this data with high power would reduce the end-to-end error probability. However, when E_s/N_0 is large, the relay is located close to the destination and, hence, increasing relay transmit power would not provide that much improvement in the R-D link and, accordingly, in the end-to-end error probability.

In order to study the effect of E_s and E_r in the optimal position of the relay node, a numerical approach is used to find d_{sr} for different setups as explained in the following two experiments. Figure 5 shows the optimal value of d_{sr}

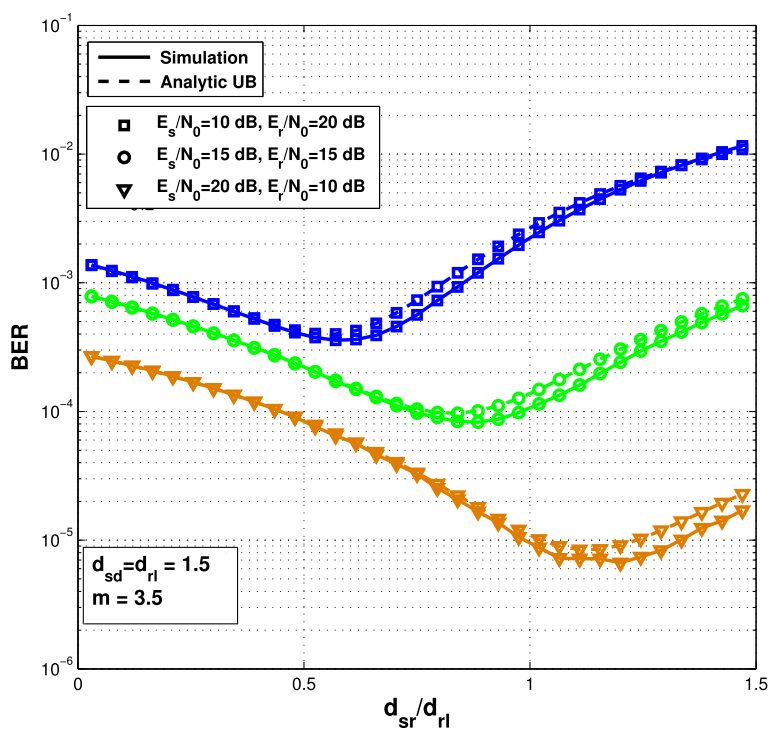

Fig. 3 Bit error rate versus d_{sr} at different values of E_s/N_0 and E_r/N_0

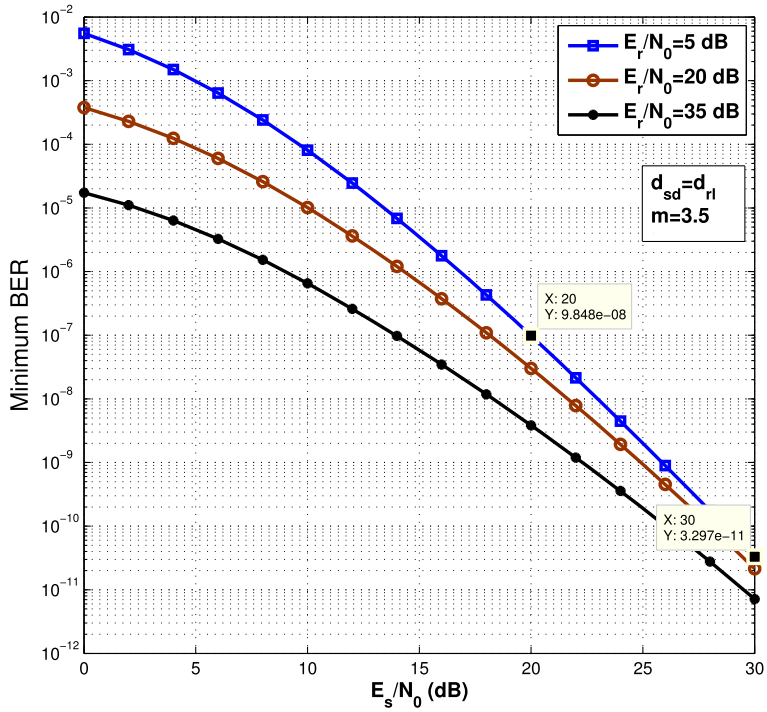

Fig. 4 Minimum (BER) at optimum d_{sr} versus E_s/N_0 at different values of E_r/N_0

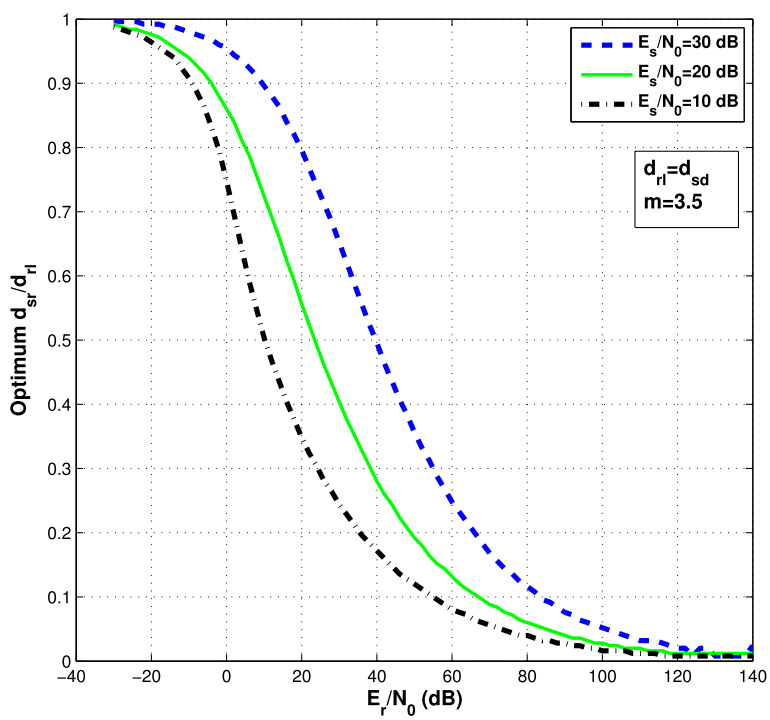

Fig. 5 Optimum relay position against E_r/N_0 at different values of E_s/N_0

against the relay transmit SNR E_r/N_0 at different values of the source transmit SNR E_s/N_0. The relay node is to be positioned on the straight line connecting the source and the destination. The length of the S-D link is $d_{sd} = 2m$ and the path loss exponent is $m = 3.5$. We find that when $E_r << E_s$, the optimal position of the relay $d_{sr} > d_{sd}/2$ (i.e., the relay has to be close to the destination node). That is because the source energy E_s is large enough such that the communication in the S-R link is reliable even when d_{sr} is large. However, the smaller value of E_r requires the positioning of the relay close to the destination to increase the receive SNR of the R-D link.

Figure 6 shows the optimum value of d_{sr} against the source transmit SNR E_s/N_0 at different values of the relay transmit SNR E_r/N_0. We find that the optimal d_{sr} increase as the source transmit SNR E_s/N_0 increase for a given value of E_r/N_0.

7 Conclusions

In this paper, we presented the MAP and the AC-MAP decoding schemes for the MISO wireless relay network that adopts the DF as a cooperation protocol. The results and discussions through the paper show that the AC-MAP is an optimal decoder as the MAP but with much less complexity. We derived a closed form expression for the upper bound on the bit error probability. The numerical results show that the upper bound expression is very tight. The derived closed form expression can be used for the solution of power allocation, partner assignment, and relay positioning problems. In this paper, we used the UB to study the optimal position of the relay node.

Appendix

In this Appendix, we derive the solution of integrations denoted by I_{11}, I_{12}, I_{21}, and I_{22}

Derivation of I_{11} and I_{12}

Let

$$I_1 = \int_0^\infty \operatorname{erfc}\left(\sqrt{x} + \frac{b}{\sqrt{x}}\right) e^{-x/\gamma} dx \tag{62}$$

Integrating (62) using integration by parts where $u = \operatorname{erfc}(\sqrt{x} + b/\sqrt{x}), dv = e^{-x/\gamma} dx, du = \frac{1}{\sqrt{\pi x}}\left(1 - \frac{b}{x}\right) e^{-(x+b)^2/x} dx$, and $v = -\gamma e^{-x/\gamma}$, yields

$$\begin{aligned} I_1 &= \left[-\gamma \operatorname{erfc}\left(\sqrt{x} + \frac{b}{\sqrt{x}}\right) e^{-x/\gamma} \right. \\ &\quad \left. - \frac{\gamma}{\sqrt{\pi}} \int \frac{1}{\sqrt{x}}\left(1 - \frac{b}{x}\right) e^{-x/\gamma - (x+b)^2/x} dx\right]_0^\infty \\ &= \begin{cases} g_1 & b \geq 0 \\ 1 + g_1 & b < 0 \end{cases} \end{aligned} \tag{63}$$

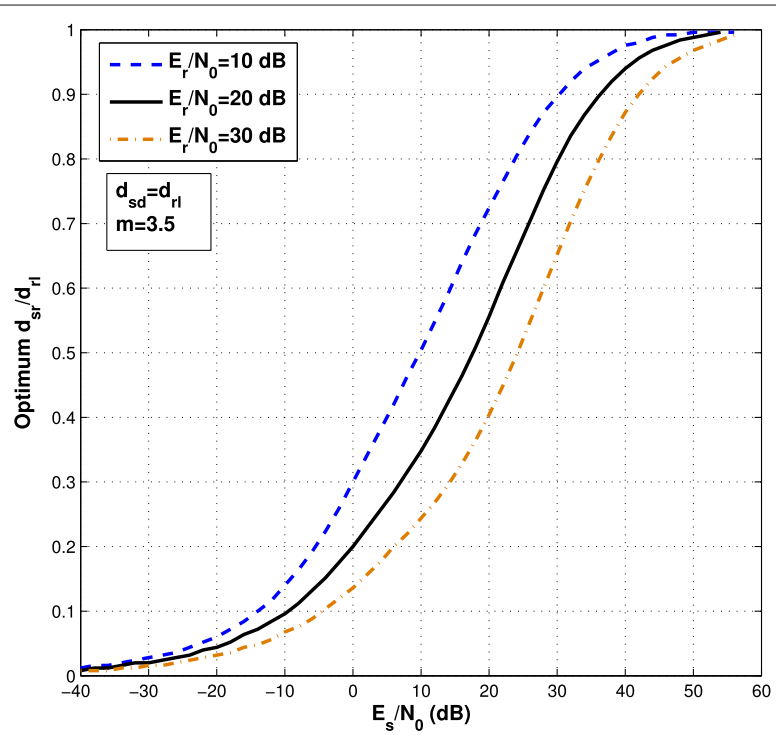

Fig. 6 Optimum relay position against E_s/N_0 at different values of E_r/N_0

where

$$g_1 = \frac{\gamma}{\sqrt{\pi}} \int_0^\infty \frac{1}{\sqrt{x}} \left(1 - \frac{b}{x}\right) e^{-x/\gamma - (x+b)^2/x} dx$$
$$= \frac{\gamma}{\sqrt{\pi}} e^{-2d-r} \int_0^\infty \frac{1}{\sqrt{x}} \left(\Gamma - \frac{d}{x}\right) e^{-x-d^2/x} dx \quad (64)$$

and $\Gamma = \sqrt{\gamma/(1+\gamma)}$, $d = b\sqrt{1+1/\gamma}$, and $r = 2b(1-1/\Gamma)$. Evaluating the integral of (64) yields

$$I_1 = \begin{cases} I_{11}(\gamma, b) & b \geq 0 \\ I_{12}(\gamma, b) & b < 0 \end{cases} \quad (65)$$

where

$$I_{11}(\gamma, b) = \frac{1}{2}(1-\Gamma)e^{-4d-r} \quad (66)$$

and

$$I_{12}(\gamma, b) = 1 - \frac{1}{2}(1+\Gamma)e^{-r} \quad (67)$$

Derivation of I_{21} and I_{22}

Let

$$I_2 = \int_0^\infty \operatorname{erfc}\left(\sqrt{x} + \frac{b}{\sqrt{x}}\right) x e^{-x/\gamma} dx \quad (68)$$

Integrating (68) using integration by parts where $u = \operatorname{erfc}(\sqrt{x} + b/\sqrt{x})$, $dv = xe^{-x/\gamma}dx$, $v = -\gamma(\gamma + x)e^{-x/\gamma}$, and $du = \frac{1}{\sqrt{\pi x}}\left(1 - \frac{b}{x}\right) e^{-(x+b)^2/x} dx$, yields

$$I_2 = \left[-\gamma(\gamma + x)\operatorname{erfc}\left(\sqrt{x} + \frac{b}{\sqrt{x}}\right) e^{-x/\gamma} - \frac{\gamma}{\sqrt{\pi}} \int \frac{1}{\sqrt{x}} \left(1 - \frac{b}{x}\right) (\gamma + x) e^{-x/\gamma - (x+b)^2/x} dx \right]_0^\infty$$
$$= \begin{cases} g_2 & b \geq 0 \\ 1 + g_2 & b < 0 \end{cases} \quad (69)$$

where

$$g_2 = \frac{\gamma}{\sqrt{\pi}} \int_0^\infty \frac{1}{\sqrt{x}} \left(1 - \frac{b}{x}\right) (\gamma + x) e^{-x/\gamma - (x+b)^2/x} dx$$
$$= \gamma I_1 - \frac{\gamma}{\sqrt{\pi}} \frac{e^{-2d-r}}{1 + 1/\gamma} \int_0^\infty \frac{1}{\sqrt{x}} x \left(\Gamma - \frac{d}{x}\right) e^{-x-d^2/x} dx \quad (70)$$

Evaluating the integral of (70) yields

$$I_2 = \begin{cases} I_{21}(\gamma, b) & b \geq 0 \\ I_{22}(\gamma, b) & b < 0 \end{cases} \quad (71)$$

where

$$I_{21}(\gamma, b) = \gamma I_{11}(\gamma, b) - \Gamma^2 e^{-4d-r} \left(\Gamma\left(\frac{1}{2} + d\right) - d\right) \quad (72)$$

Substituting for the values of d and r into (72) and performing some mathematical manipulations yields

$$I_{21}(\gamma, b) = \left(\frac{\gamma}{2}(1-\Gamma) - \Gamma^2\left(\frac{\Gamma}{2} - \frac{b(1-\Gamma)}{\Gamma}\right)\right) \times e^{-2b(1+\Gamma)/\Gamma} \quad (73)$$

$$I_{22}(\gamma, b) = \gamma I_{12}(\gamma, b) - \Gamma^2 e^{-r}\left(\Gamma\left(\frac{1}{2} - d\right) - d\right) \quad (74)$$

Substituting for the values of d and r into (72) and performing some mathematical manipulations yields

$$I_{22}(\gamma, b) = \gamma - \left(\frac{\gamma}{2}(1+\Gamma) + \Gamma^2\left(\frac{\Gamma}{2} - \frac{b(1-\Gamma)}{\Gamma}\right)\right) \times e^{2b(1-\Gamma)/\Gamma} \quad (75)$$

Abbreviations

AC: Alamouti combining; AF: Amplify and forward; ARQ: Automatic repeat request; AWGN: Additive white Gaussian noise; BER: Bit error rate; BPSK: Binary phase shift keying; DF: Decode and forward; EGC: Equal gain combining; MAP: Maximum a posterior; MISO: Multiple input single output; ML: Maximum likelihood; MRC: Maximum ratio combining; SC: Selection combining; SNR: Signal-to-noise ratio; STBC: Space time block code; UB: Upper bound

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Authors' contributions

TAK proposed the ideas and outlines for theoretical analysis, reviewed the analysis and numerical results, and revised and prepared the final manuscript. HM performed the theoretical analysis and simulations and prepared the first version of the manuscript.Both authors have read and approved the final manuscript.

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