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# Two-hop cognitive DF relaying with wireless power transfer in time and power domains

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## Abstract

In this paper, we consider a wireless powered cognitive relaying system with a secondary relay (SR) capable of harvesting wireless energy. Along with an access point (AP) continuously transmitting the primary data to a primary user (PU), a secondary source (SS) can transmit the secondary data to a secondary destination (SD) with the help of SR using the decode-and-forward (DF) protocol. SR can harvest energy from both SS and AP in both time and power domains using time-splitting and power-splitting techniques. The interference from primary data transmissions can help boost the amount of harvested energy at SR. The transmit power of SS is regulated by the interference threshold at PU and the allowable peak power. Despite the above two constraints, the transmit power of SR is further constrained by the amount of harvested energy. Once SR successfully decodes the data from SS, it will forward the data to SD using a constrained power. We analyze the approximate outage probabilities for both primary and secondary systems. Simulation results are provided to verify the effectiveness of our theoretical analysis and reveal the impacts of various parameters to the outage performance.

**Keywords:** Cognitive relaying, Spectrum sharing, Decode-and-forward, Outage probability, Energy harvesting, Wireless power transfer

## 1 Introduction

Spectrum sharing between primary users (PUs) and secondary users (SUs) can promisingly alleviate the spectrum scarcity problem [1]. SUs may share the spectrum with PUs in separate time [2], disjoint bandwidth [3], and space domain [4], etc. In underlay scenario, SUs can simultaneously access the licensed spectrum with PUs, provided that SUs do not violate the interference constraint at the PU receiver [5]. Cooperative relaying can improve the communication robustness by achieving the space diversity, but it degrades the spectrum efficiency due to the half-duplex operation of relays, which can be tackled by using the non-orthogonal multiple access (NOMA) and network coding techniques [6]. Cooperative relaying in cognitive radio networks can not only improve coverage for SUs' transmissions, but also offer better reliability compared with the point-to-point link [7–9]. The achievable capacity and the outage probability of a secondary link assisted by a decode-and-forward (DF) relay were studied in [10]. Duong et al. studied the DF cognitive relaying (CR) with multiple primary transceivers, where the transmit powers

of secondary source and relay should be properly set under the interference constraints of all the primary transceivers [11]. The effect of interference constraint on the diversity gain in DF relaying networks with the the best-relay selection was studied in [12]. Xia et al. analyzed the outage probability of amplify-and-forward (AF) CR under average interference constraints [13]. Zhai et al. enabled either PU or SU receivers to perform the successive interference cancelation (SIC) to improve the throughput for the DF-CR [14]. The optimal power allocation in an AF-CR network with either single-relay or multi-relay was studied in [15] to maximize the secondary signal-to-noise ratio (SNR) while keeping the interference to PU below a threshold. Majhi et al. studied the AF-CR with the best relay selection, and analyzed the secondary outage probability by considering both direct and cooperative links [16].

Wireless energy harvesting (EH) has been intensively studied nowadays, as the radio frequency (RF) signal can carry not only information but also energy [17] and the RF-EH is more controllable to satisfy the communication requirements [18]. Wireless power transfer (WPT) can help charge the energy-limited nodes using the RF signal. Since the energy harvester and the information decoder usually work with separate circuits, time-switching (TS) and power-splitting (PS) techniques were designed to realize EH and information decoding (ID) in time and power domains, respectively [19]. Both the TS-based relaying (TSR) and the PS-based relaying (PSR) were proposed in [20], where a fraction of time or power is allocated for the EH at a relay, and it is shown that the PSR results in higher throughput at high SNR regime. Nasir et al. also proposed a TS-based wireless EH protocol and properly modeled the status of energy accumulation at the relay with continuous and discrete time EH [21]. Modem et al. compared the performance of TSR and PSR in two-hop networks with a battery-assisted EH-AF relay, where the EH relay augments the harvested energy with energy drawn from a battery [22]. A hybrid EH protocol was proposed in [23] by combining both TS and PS methods. Two relays were adopted to alternately harvest energy and forward data from source to destination in [24]. Similarly, two ground nodes can harvest energy from two unmanned aerial vehicles (UAV) and alternately send their data to the corresponding UAV [25]. When both source and relay work with EH, a power beacon can be exploited to wireless charge the two nodes for the cooperative relaying [26].

The EH capability has been integrated into cognitive radio networks [27]. For the case that SUs harvest energy from PUs, the throughput was optimized under outage and interference constraints of PUs [28]. Furthermore, a SU can cooperatively forward the primary data and transmit its own data simultaneously over orthogonal subcarriers by using the harvested energy [29]. When a secondary relay (SR) can harvest energy from secondary source (SS) and forward data to secondary destination (SD) under the interference constraint of PU, the outage performance was studied in [30]. The authors of [31] studied the energy efficiency and rate-energy tradeoff by jointly optimizing PS factors and power allocation at relay. Energy-limited SUs can harvest energy from not only the received information signal but also the co-channel interference [32]. If there are multiple primary transceivers, both SS and SR can harvest energy from multiple PUs' transmission, while the transmit power of SR should be strictly controlled under the peak interference constraint of each PU receiver [33]. SR can harvest energy from the secondary signals as well as the primary interference by using the TS policy [34, 35]. The energy

harvesting and accumulation is properly modeled in [36] for the CR. Furthermore, the underlay framework can be extended to a large-scale Internet of things (IoT) network in [37]. As studied in [38, 39], SUs can wireless power energy-limited PUs to exchange for more opportunities of spectrum sharing.

For the typical wireless powered communication networks, such as passive radio frequency identification (RFID) or sensor networks, wireless devices are powered only by WPT and transmit their data using the harvested energy [40]. The harvested energy is often transiently kept by a supercapacitor which has the characteristics of small form factor and fast charge–discharge [41]. Due to the leakage of supercapacitor and the absence of energy storage or management, the possible remaining energy or the harvested energy after transmission cannot be used in the next communication cycle. We consider this energy consumption model in the EH based CR process. In the traditional EH based CR schemes, SR often sets its transmit power by considering two constraints among the PU interference threshold, the available energy, and the allowable peak power. When the power of SS is dynamically adjusted, the analysis becomes more difficult, as the available energy of SR depends directly on the transmit power of SS. In order to make the protocol more meaningful and reveal the complex relationship between users, we consider all the three constraints for the continuous power setting at SR.

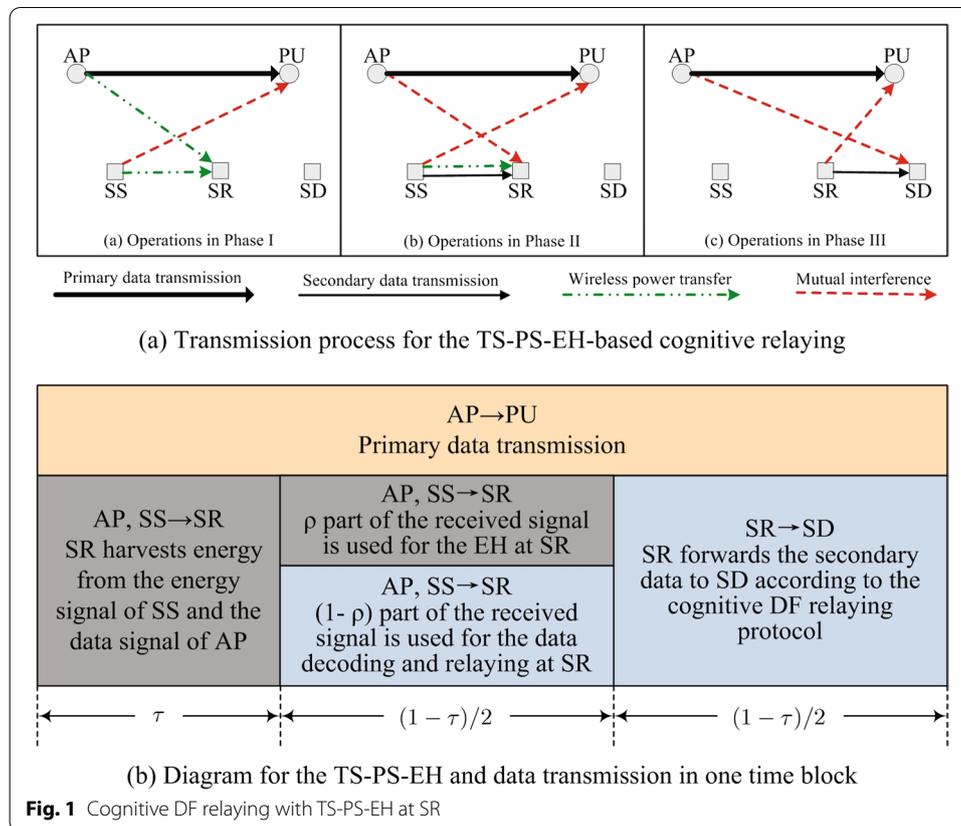
In this work, we consider a wireless powered CR system, wherein SS transmits the secondary data to SD with assistance from an EH-based SR operating in the DF fashion. In the primary system, an access point (AP) continuously transmits the primary data to a PU. In the EH phase, SR can harvest energy from both SS and AP in both time and power domains by jointly using TS and PS techniques. The interference from AP can help boost the amount of harvested energy at SR. In the data transmission phase, the transmit power of SS is constrained by the PU interference threshold and the allowable peak power, while the transmit power of SR is constrained by the PU interference threshold, the available energy, and the allowable peak power. By considering various powers of SS and SR, as well as the mutual interference between different users, we derive the approximate outage probabilities for both primary and secondary systems. Through extensive simulations, we study the impacts of various parameters on the outage performance, such as TS and PS factors, transmission rate, distance between the two systems, and distance between SUs, etc.

The rest of this paper is organized as follows: Sect. 2 illustrates the methods. Section 3 analyzes the success probability of SR decoding. Sections 4 and 5 analyze the outage probabilities of primary and secondary systems, respectively. Results and discussion are presented in Sect. 6. Section 7 concludes this work.

## 2 Methods

### 2.1 Cognitive radio network

We consider a cognitive radio network as shown in Fig. 1, where AP continuously transmits the primary data to PU, and SR cooperatively forwards the secondary data from SS to SD. In the underlay paradigm, along with the AP transmission, SS or SR can concurrently transmit data under the constraint that the interference caused to PU should not exceed an interference threshold  $I_0$ . By overhearing pilot or control signals broadcast by PU, both SS and SR can estimate the channel fading between themselves and PU for the



interference control. Each node is assumed to work in the half-duplex mode by using one omnidirectional antenna. Throughout this paper, we use the subscripts a, p, s, r, and d to denote AP, PU, SS, SR, and SD, respectively. We use  $h_{xy}$  to denote the small-scale channel fading from x to y with  $x, y \in \{p, a, s, r, d\}$ . Each channel is assumed to undergo independent Rayleigh block fading, which changes independently across different links and time blocks. The small-scale power gain  $G_{xy} = |h_{xy}|^2$  is exponentially distributed with unit mean. Apart from the small-scale channel fading, the signal also suffers from the large-scale path-loss modeled as  $\ell_{xy}^{1/2} = d_{xy}^{-\alpha/2}$ , where  $\alpha$  is the path-loss exponent and  $d_{xy}$  is the distance between x and y.

We assume that AP, PU, SS, and SD have stable power supplies, while SR has no continuous power supplies. SR can harvest energy from the RF signals broadcast by AP and SS by jointly using TS and PS techniques. SR may represent a wireless sensor node, which is deployed in the extreme areas and the battery recharge or replacement is difficult. In practice, the super-capacitor is a suitable energy device for EH sensor nodes, as it can withstand rapid charging and discharging cycles [42]. The interference from AP becomes a useful energy source that can help boost the amount of harvested energy at SR. The transmit power of SR may be high or low due to the random variation of channel fading, so it is necessary to consider the interference constraint at PU.

In the observed transmission period, the whole time is divided into equal-length blocks with each block normalized as one second. In each block, AP keeps on transmitting the primary data to PU without interruption, while SS and SR concurrently perform the EH-based

cognitive DF relaying. As shown in Fig. 1, each time block consists of three phases, wherein Phase I occupies  $\tau$  fraction of time for the EH, and each of the remaining two phases lasts for  $\frac{1-\tau}{2}$  fraction of time for the CR. SR harvests RF energy in both time and power domains using jointly TS and PS techniques, and forwards data in the cognitive DF relaying manner. Since the transmit powers of SS and SR are strictly constrained and the channel may suffer from deep fading or blockage, we assume there is no direct link between SS and SD, so the data can reach SD only with the assistance of SR.

### 2.2 Energy harvesting of SR

In Phase I, along with AP transmitting the primary data to PU, SS transmits the predefined energy signal to SR by using the maximal power. SR can harvest energy not only from the energy signal sent by SS, but also from the data signal sent by AP. The interference from AP is utilized as an energy source for SR to boost its EH amount. SR saves all the harvested energy into its battery. Due to the independence between the energy signal from SS and the data signal from AP, the amount of energy harvested by SR in Phase I is denoted as  $E_{eh}^I$  and given by

$$E_{eh}^I = \tau \eta (P_a G_{ar} \ell_{ar} + \hat{P}_s G_{sr} \ell_{sr}), \tag{1}$$

where  $\eta$  denotes the energy conversion efficiency,  $P_a$  and  $\hat{P}_s$  represent the transmit power of AP and the peak power of SS, respectively.

In Phase II, along with AP transmitting the primary data to PU, SS transmits the secondary data to SR. The transmit power of SS is denoted as  $P_s$  and given by

$$P_s = \min \left( \frac{I_0}{G_{sp} \ell_{sp}}, \hat{P}_s \right), \tag{2}$$

where the first term represents that the interference caused to PU by the SS transmission should not exceed an interference threshold  $I_0$ .

After receiving the composite signals In Phase II, SR splits  $\rho$  fraction for the EH and  $(1 - \rho)$  fraction for the data decoding. We assume that the secondary data transmitted by SS and the primary data transmitted by AP are independent. Then, the amount of energy harvested by SR in Phase II is denoted as  $E_{eh}^{II}$  and given by

$$E_{eh}^{II} = \frac{1 - \tau}{2} \rho \eta (P_a G_{ar} \ell_{ar} + P_s G_{sr} \ell_{sr}). \tag{3}$$

The total amount of energy harvested by SR in both Phases I and II is denoted as  $E_{eh}$  and given by

$$E_{eh} = E_{eh}^I + E_{eh}^{II}. \tag{4}$$

With the block fading assumption, the channel fading coefficient keeps invariant in each whole time block, so EH and CR are mutually dependent.

### 2.3 Data transmissions

#### 2.3.1 Transmissions in Phase I

We assume that the energy signal is a priori-known by PU and the channel state information of the link SS → PU is available. In Phase I, after receiving the composite signals, PU could cancel the energy signal of SS and then decode the data of AP. The achievable rate of the link AP → PU is denoted as  $C_{ap}^I$  and given by

$$C_{ap}^I = \tau \log_2 \left( 1 + \gamma_{ap}^I \right), \tag{5}$$

where  $\gamma_{ap}^I = \frac{P_a G_{ap} \ell_{ap}}{N_0}$ . At each receiver, the total power of additive white Gaussian noise (AWGN) is denoted as  $N_0 = \sigma_{na}^2 + \sigma_{nc}^2$ , where  $\sigma_{na}^2$  represents the power of AWGN at the receiving antenna and  $\sigma_{nc}^2$  represents the power of AWGN due to the conversion from RF signal to baseband signal [19].

#### 2.3.2 Transmissions in Phase II

In Phase II, SS transmits the secondary data to SR, and AP continues to transmit the primary data to PU. By treating the received secondary data as interference, the achievable rate of primary data at PU is denoted as  $C_{ap}^{II}$  and given by

$$C_{ap}^{II} = \frac{1 - \tau}{2} \log_2 \left( 1 + \gamma_{ap}^{II} \right), \tag{6}$$

where  $\gamma_{ap}^{II} = \frac{P_a G_{ap} \ell_{ap}}{P_s G_{sp} \ell_{sp} + N_0}$ .

In Phase II, SR splits  $\rho$  fraction of the received composite signals for the EH, and decodes data based on the remaining  $(1 - \rho)$  fraction. By treating the primary data as interference, the achievable rate of the secondary data at SR can be denoted as  $C_{sr}^{II}$  and given by

$$C_{sr}^{II} = \frac{1 - \tau}{2} \log_2 \left( 1 + \gamma_{sr}^{II} \right), \tag{7}$$

where  $\gamma_{sr}^{II} = \frac{(1-\rho)P_s G_{sr} \ell_{sr}}{(1-\rho)P_a G_{ar} \ell_{ar} + (1-\rho)\sigma_{na}^2 + \sigma_{nc}^2} = \frac{P_s G_{sr} \ell_{sr}}{P_a G_{ar} \ell_{ar} + \sigma_{na}^2 + \frac{\sigma_{nc}^2}{1-\rho}}$ .

#### 2.3.3 Transmissions in Phase III

If SR correctly decodes the secondary data in Phase II, it will forward the secondary data to SD in Phase III. The transmit power of SR is denoted as  $P_r$  and given by

$$P_r = \min \left( \frac{I_0}{G_{rp} \ell_{rp}}, \hat{P}_r, \frac{E_{eh}}{(1 - \tau)/2} \right), \tag{8}$$

where the first term is obtained under the interference constraint at PU,  $\hat{P}_r$  represents the peak power of SR, and the last term represents the available power of SR by depleting its battery.

By treating the received secondary data as interference, the achievable rate of primary data at PU is denoted as  $C_{ap}^{III}$  and given by

$$C_{ap}^{III} = \frac{1 - \tau}{2} \log_2 \left( 1 + \gamma_{ap}^{III} \right), \tag{9}$$

where  $\gamma_{ap}^{III} = \frac{P_a G_{ap} \ell_{ap}}{P_r G_{rp} \ell_{rp} + N_0}$ .

If SR does not forward data to SD in Phase III, no interference is caused to PU. In this sense, the achievable rate of primary data at PU is denoted as  $\tilde{C}_{ap}^{III}$  and given by

$$\tilde{C}_{ap}^{III} = \frac{1 - \tau}{2} \log_2 \left( 1 + \tilde{\gamma}_{ap}^{III} \right), \tag{10}$$

where  $\tilde{\gamma}_{ap}^{III} = \frac{P_a G_{ap} \ell_{ap}}{N_0}$ .

When SR forwards the secondary data to SD in Phase III, the achievable rate at SD is denoted as  $C_{rd}^{III}$  and given by

$$C_{rd}^{III} = \frac{1 - \tau}{2} \log_2 \left( 1 + \gamma_{rd}^{III} \right), \tag{11}$$

where  $\gamma_{rd}^{III} = \frac{P_r G_{rd} \ell_{rd}}{P_a G_{ad} \ell_{ad} + N_0}$ .

### 3 Success probability of SR decoding

We assume that the secondary data is transmitted with fixed rate  $\nu_s$ . The secondary data transmission is assumed to be successful when the channel achievable rate is greater than  $\nu_s$ , otherwise, the transmission fails. Let  $\mathcal{D}$  denote the event that SR correctly decodes the secondary data in Phase II. The success probability of SR decoding can be expressed as  $\Pr \{ \mathcal{D} \} = \Pr \{ C_{sr}^{II} \geq \nu_s \}$ . Since the transmit power of SS is either  $\frac{I_0}{G_{sp} \ell_{sp}}$  or  $\hat{P}_s$  according to (2), we can have

$$\Pr \{ \mathcal{D} \} = \underbrace{\Pr \left\{ C_{sr}^{II} \geq \nu_s, G_{sp} > \frac{I_0}{\hat{P}_s \ell_{sp}} \right\}}_{\Pr \{ \mathcal{D}_a \}} + \underbrace{\Pr \left\{ C_{sr}^{II} \geq \nu_s, G_{sp} \leq \frac{I_0}{\hat{P}_s \ell_{sp}} \right\}}_{\Pr \{ \mathcal{D}_b \}}. \tag{12}$$

The event  $\mathcal{D}_a$  represents the successful decoding at SR when the transmit power of SS is  $P_s = \frac{I_0}{G_{sp} \ell_{sp}}$ , which means  $G_{sp} > \frac{I_0}{\hat{P}_s \ell_{sp}}$ . The event  $\mathcal{D}_b$  represents the successful decoding at SR when the transmit power of SS is  $P_s = \hat{P}_s$ , which means  $G_{sp} \leq \frac{I_0}{\hat{P}_s \ell_{sp}}$ .

The first probability of (12) can be derived as

$$\begin{aligned} \Pr \{ \mathcal{D}_a \} &= \int_{\frac{I_0}{\hat{P}_s \ell_{sp}}}^{\infty} \frac{I_0 \ell_{sr}}{I_0 \ell_{sr} + \xi_s P_a \ell_{ar} \ell_{sp} G_{sp}} \\ &\times \exp \left\{ - \left[ 1 + \frac{\xi_s \left( \sigma_{na}^2 + \frac{\sigma_{nc}^2}{1 - \rho} \right) \ell_{sp}}{I_0 \ell_{sr}} \right] G_{sp} \right\} dG_{sp}, \end{aligned} \tag{13}$$

where  $\xi_s = 2^{\frac{2\nu_s}{1 - \tau}} - 1$ .

The second probability of (12) can be derived as

$$\begin{aligned} \Pr \{ \mathcal{D}_b \} &= \frac{\hat{P}_s \ell_{sr}}{\hat{P}_s \ell_{sr} + \xi_s P_a \ell_{ar}} \exp \left[ - \frac{\xi_s \left( \sigma_{na}^2 + \frac{\sigma_{nc}^2}{1 - \rho} \right)}{\hat{P}_s \ell_{sr}} \right] \\ &\times \left[ 1 - \exp \left( - \frac{I_0}{\hat{P}_s \ell_{sp}} \right) \right]. \end{aligned} \tag{14}$$

#### 4 Outage probability of the primary system

We assume that the primary data is transmitted with fixed rate  $\nu_p$ . The primary data transmission is assumed to be successful when the channel achievable rate is greater than  $\nu_p$ . Otherwise, the transmission fails. Since AP continuously transmits the primary data to PU in all the three phases, denoted as  $P_{\text{outp}}$ , the average outage probability can be defined as

$$P_{\text{outp}} = \tau P_{\text{outp}}^{\text{I}} + \frac{1 - \tau}{2} P_{\text{outp}}^{\text{II}} + \frac{1 - \tau}{2} P_{\text{outp}}^{\text{III}}, \tag{15}$$

where  $P_{\text{outp}}^{\text{I}}$ ,  $P_{\text{outp}}^{\text{II}}$ , and  $P_{\text{outp}}^{\text{III}}$  represent the outage probabilities of primary data transmissions in Phases I, II, and III, respectively.

The outage probability of the primary data transmission in Phase I can be expressed as  $P_{\text{outp}}^{\text{I}} = \Pr \left\{ C_{\text{ap}}^{\text{I}} < \tau \nu_p \right\}$  and derived as

$$P_{\text{outp}}^{\text{I}} = 1 - \exp \left( - \frac{\xi_p N_0}{P_a \ell_{\text{ap}}} \right), \tag{16}$$

where  $\xi_p = 2^{\nu_p} - 1$ .

The outage probability of the primary data transmission in Phase II can be expressed as  $P_{\text{outp}}^{\text{II}} = \Pr \left\{ C_{\text{ap}}^{\text{II}} < \frac{1 - \tau}{2} \nu_p \right\}$  and derived as

$$P_{\text{outp}}^{\text{II}} = 1 - \frac{1}{P_a \ell_{\text{ap}} + \xi_p \hat{P}_s \ell_{\text{sp}}} \exp \left( - \frac{\xi_p N_0}{P_a \ell_{\text{ap}}} \right) \times \left[ \xi_p \hat{P}_s \ell_{\text{sp}} \exp \left( - \frac{I_0}{\hat{P}_s \ell_{\text{sp}}} - \frac{\xi_p I_0}{P_a \ell_{\text{ap}}} \right) + P_a \ell_{\text{ap}} \right], \tag{17}$$

where we consider that the transmit power of SS is either  $\frac{I_0}{G_{\text{sp}} \ell_{\text{sp}}}$  or  $\hat{P}_s$  as per (2).

Considering SR forwards the secondary data or not, the outage probability of the primary data transmission in Phase III can be expressed as

$$P_{\text{outp}}^{\text{III}} = \Pr \left\{ \mathcal{D}, C_{\text{ap}}^{\text{III}} < \frac{1 - \tau}{2} \nu_p \right\} + \Pr \left\{ \bar{\mathcal{D}}, \tilde{C}_{\text{ap}}^{\text{III}} < \frac{1 - \tau}{2} \nu_p \right\}, \tag{18}$$

where  $\bar{\mathcal{D}}$  is the complement event of  $\mathcal{D}$ . Due to the independence between  $\bar{\mathcal{D}}$  and the interference-free primary transmission, the second probability of (18) equals  $\Pr \{ \bar{\mathcal{D}} \} \Pr \left\{ \tilde{C}_{\text{ap}}^{\text{III}} < \frac{1 - \tau}{2} \nu_p \right\}$ , where  $\Pr \{ \bar{\mathcal{D}} \} = 1 - \Pr \{ \mathcal{D} \}$  with  $\Pr \{ \mathcal{D} \}$  analyzed in (12) and  $\Pr \left\{ \tilde{C}_{\text{ap}}^{\text{III}} < \frac{1 - \tau}{2} \nu_p \right\}$  has the same result as (16).

Since the event  $\mathcal{D}$  consists of two uncorrelated events  $\mathcal{D}_a$  and  $\mathcal{D}_b$ , the first probability of (18) can be analyzed as follows:

$$\begin{aligned}
 \Pr \left\{ \mathcal{D}, C_{\text{ap}}^{\text{III}} < \frac{1-\tau}{2} v_p \right\} &\approx \Pr \{ \mathcal{D}_a \} \underbrace{\Pr \left\{ C_{\text{ap}}^{\text{III}} < \frac{1-\tau}{2} v_p \mid P_s = \frac{I_0}{G_{\text{sp}} \ell_{\text{sp}}} \right\}}_{p_{\text{outp}}^{\text{a11}} + p_{\text{outp}}^{\text{a12}} + p_{\text{outp}}^{\text{a2}} + p_{\text{outp}}^{\text{a3}}} \\
 &+ \Pr \{ \mathcal{D}_b \} \underbrace{\Pr \left\{ C_{\text{ap}}^{\text{III}} < \frac{1-\tau}{2} v_p \mid P_s = \hat{P}_s \right\}}_{p_{\text{outp}}^{\text{b11}} + p_{\text{outp}}^{\text{b12}} + p_{\text{outp}}^{\text{b2}} + p_{\text{outp}}^{\text{b3}}}.
 \end{aligned} \tag{19}$$

Due to the channel block fading, the three-phase transmissions are intertwined through EH and mutual interference, which makes the analysis much involved. We strip the events  $\mathcal{D}_a$  and  $\mathcal{D}_b$  out from the joint probability, and their probabilities have been derived in (13) and (14), respectively. The accuracy of this approximation will be verified through simulations. Since SS may transmit with different powers in Phase II as shown in (2), we have two general cases, i.e., Case A and Case B. For each general case, considering different powers of SR given by (8), we have four subcases, so there are totally eight subcases as mentioned above, i.e., Case A.1.1, Case A.1.2, Case A.2, Case A.3, Case B.1.1, Case B.1.2, Case B.2, and Case B.3.

#### 4.1 Outage probabilities in Case A

In Case A, the transmit power of SS in Phase II is  $P_s = \frac{I_0}{G_{\text{sp}} \ell_{\text{sp}}}$ , we have

$$P_r = \min \left( \frac{I_0}{G_{\text{tp}} \ell_{\text{tp}}}, \hat{P}_r, \underbrace{\left( \frac{2\tau}{1-\tau} + \rho \right) \eta P_a G_{\text{ar}} \ell_{\text{ar}} + \left( \frac{2\tau}{1-\tau} \hat{P}_s + \frac{\rho I_0}{G_{\text{sp}} \ell_{\text{sp}}} \right) \eta G_{\text{sr}} \ell_{\text{sr}}}_{Y(G_{\text{ar}}, G_{\text{sp}}, G_{\text{sr}})} \right). \tag{20}$$

The cumulative distribution function (CDF) of  $Y(G_{\text{ar}}, G_{\text{sp}}, G_{\text{sr}})$  is  $\Pr \{ Y(G_{\text{ar}}, G_{\text{sp}}, G_{\text{sr}}) \leq y \}$ , derived as

$$\begin{aligned}
 F_Y(y) &= 1 - \int_0^\infty \frac{\exp(-G_{\text{sp}})}{\left( \frac{2\tau}{1-\tau} \hat{P}_s + \frac{\rho I_0}{G_{\text{sp}} \ell_{\text{sp}}} \right) \eta \ell_{\text{sr}} - \left( \frac{2\tau}{1-\tau} + \rho \right) \eta P_a \ell_{\text{ar}}} \\
 &\times \left\{ \left( \frac{2\tau}{1-\tau} \hat{P}_s + \frac{\rho I_0}{G_{\text{sp}} \ell_{\text{sp}}} \right) \eta \ell_{\text{sr}} \exp \left[ -\frac{y}{\left( \frac{2\tau}{1-\tau} \hat{P}_s + \frac{\rho I_0}{G_{\text{sp}} \ell_{\text{sp}}} \right) \eta \ell_{\text{sr}}} \right] \right. \\
 &\left. - \left( \frac{2\tau}{1-\tau} + \rho \right) \eta P_a \ell_{\text{ar}} \exp \left[ -\frac{y}{\left( \frac{2\tau}{1-\tau} + \rho \right) \eta P_a \ell_{\text{ar}}} \right] \right\} dG_{\text{sp}}.
 \end{aligned} \tag{21}$$

The probability density function (PDF) of  $Y(G_{\text{ar}}, G_{\text{sp}}, G_{\text{sr}})$  is  $\frac{dF_Y(y)}{dy}$ , derived as

$$\begin{aligned}
 f_Y(y) &= \int_0^\infty \frac{\exp(-G_{\text{sp}})}{\left( \frac{2\tau}{1-\tau} \hat{P}_s + \frac{\rho I_0}{G_{\text{sp}} \ell_{\text{sp}}} \right) \eta \ell_{\text{sr}} - \left( \frac{2\tau}{1-\tau} + \rho \right) \eta P_a \ell_{\text{ar}}} \\
 &\times \left\{ \exp \left[ -\frac{y}{\left( \frac{2\tau}{1-\tau} \hat{P}_s + \frac{\rho I_0}{G_{\text{sp}} \ell_{\text{sp}}} \right) \eta \ell_{\text{sr}}} \right] - \exp \left[ -\frac{y}{\left( \frac{2\tau}{1-\tau} + \rho \right) \eta P_a \ell_{\text{ar}}} \right] \right\} dG_{\text{sp}}.
 \end{aligned} \tag{22}$$

**4.1.1 Case A.1**

If  $\frac{I_0}{G_{rp}\ell_{rp}} < \hat{P}_r$  and  $\frac{I_0}{G_{rp}\ell_{rp}} < Y(G_{ar}, G_{sp}, G_{sr})$ , we have  $P_r = \frac{I_0}{G_{rp}\ell_{rp}}$ . We have two conditions to represent the occurrence of this case:

$$\begin{aligned} \text{Case A.1.1: } P_s &= \frac{I_0}{G_{sp}\ell_{sp}}, P_r = \frac{I_0}{G_{rp}\ell_{rp}}, \\ G_{rp} &> \frac{I_0}{\hat{P}_r\ell_{rp}}, Y(G_{ar}, G_{sp}, G_{sr}) > \hat{P}_r, \end{aligned} \tag{23}$$

$$\begin{aligned} \text{Case A.1.2: } P_s &= \frac{I_0}{G_{sp}\ell_{sp}}, P_r = \frac{I_0}{G_{rp}\ell_{rp}}, \\ G_{rp} &> \frac{I_0}{Y(G_{ar}, G_{sp}, G_{sr})\ell_{rp}}, Y(G_{ar}, G_{sp}, G_{sr}) < \hat{P}_r. \end{aligned} \tag{24}$$

Based on Case A.1.1, the probability  $P_{\text{outp}}^{\text{a11}}$  in (19) can be derived as

$$P_{\text{outp}}^{\text{a11}} = \left[ 1 - F_Y(\hat{P}_r) \right] \exp\left(-\frac{I_0}{\hat{P}_r\ell_{rp}}\right) \left\{ 1 - \exp\left[-\frac{\xi_p(I_0 + N_0)}{P_a\ell_{ap}}\right] \right\}, \tag{25}$$

where  $F_Y(\hat{P}_r)$  can be calculated according to (21) by replacing  $y$  with  $\hat{P}_r$ .

Based on Case A.1.2, the probability  $P_{\text{outp}}^{\text{a12}}$  in (19) can be derived as

$$P_{\text{outp}}^{\text{a12}} = \left\{ 1 - \exp\left[-\frac{\xi_p(I_0 + N_0)}{P_a\ell_{ap}}\right] \right\} \int_0^{\hat{P}_r} \exp\left(-\frac{I_0}{y\ell_{rp}}\right) f_Y(y) dy, \tag{26}$$

where  $f_Y(y)$  is given in (22).

**4.1.2 Case A.2**

If  $\hat{P}_r < \frac{I_0}{G_{rp}\ell_{rp}}$  and  $\hat{P}_r < Y(G_{ar}, G_{sp}, G_{sr})$ , we have  $P_r = \hat{P}_r$ . We have one condition to represent the occurrence of this case:

$$\begin{aligned} \text{Case A.2: } P_s &= \frac{I_0}{G_{sp}\ell_{sp}}, P_r = \hat{P}_r, \\ G_{rp} &< \frac{I_0}{\hat{P}_r\ell_{rp}}, Y(G_{ar}, G_{sp}, G_{sr}) > \hat{P}_r. \end{aligned} \tag{27}$$

Based on Case A.2, the probability  $P_{\text{outp}}^{\text{a2}}$  in (19) can be derived as

$$\begin{aligned} P_{\text{outp}}^{\text{a2}} &= \left\{ 1 - \exp\left(-\frac{I_0}{\hat{P}_r\ell_{rp}}\right) - \frac{P_a\ell_{ap}}{P_a\ell_{ap} + \xi_p\hat{P}_r\ell_{rp}} \exp\left(-\frac{\xi_p N_0}{P_a\ell_{ap}}\right) \right. \\ &\quad \left. \times \left[ 1 - \exp\left(-\frac{I_0}{\hat{P}_r\ell_{rp}} - \frac{\xi_p I_0}{P_a\ell_{ap}}\right) \right] \right\} \left[ 1 - F_Y(\hat{P}_r) \right], \end{aligned} \tag{28}$$

where  $F_Y(\hat{P}_r)$  can be calculated according to (21) by replacing  $y$  with  $\hat{P}_r$ .

**4.1.3 Case A.3**

If  $Y(G_{ar}, G_{sp}, G_{sr}) < \frac{I_0}{G_{rp}\ell_{rp}}$  and  $Y(G_{ar}, G_{sp}, G_{sr}) < \hat{P}_r$ , we have  $P_r = Y(G_{ar}, G_{sp}, G_{sr})$ . We have one condition to represent the occurrence of this case:

$$\begin{aligned} \text{Case A.3: } P_s &= \frac{I_0}{G_{sp}\ell_{sp}}, P_r = Y(G_{ar}, G_{sp}, G_{sr}), \\ G_{rp} &< \frac{I_0}{Y(G_{ar}, G_{sp}, G_{sr})\ell_{rp}}, Y(G_{ar}, G_{sp}, G_{sr}) < \hat{P}_r. \end{aligned} \tag{29}$$

Based on Case A.3, the probability  $P_{\text{outp}}^{\text{a3}}$  in (19) can be derived as

$$\begin{aligned} P_{\text{outp}}^{\text{a3}} &= \int_0^{\hat{P}_r} \left\{ 1 - \exp\left(-\frac{I_0}{y\ell_{rp}}\right) - \frac{P_a\ell_{ap}}{P_a\ell_{ap} + \xi_p\ell_{rp}y} \exp\left(-\frac{\xi_p N_0}{P_a\ell_{ap}}\right) \right. \\ &\quad \left. \times \left[ 1 - \exp\left(-\frac{\xi_p N_0}{P_a\ell_{ap}} - \frac{I_0}{y\ell_{rp}}\right) \right] \right\} f_Y(y) dy, \end{aligned} \tag{30}$$

where  $f_Y(y)$  is given in (22).

**4.2 Outage probabilities in Case B**

When  $G_{sp} \leq \frac{I_0}{\hat{P}_s\ell_{sp}}$ , the transmit power of SS in Phase II is  $P_s = \hat{P}_s$ , we have

$$P_r = \min \left( \frac{I_0}{G_{rp}\ell_{rp}}, \hat{P}_r, \underbrace{\left( \frac{2\tau}{1-\tau} + \rho \right) \eta \left( P_a G_{ar} \ell_{ar} + \hat{P}_s G_{sr} \ell_{sr} \right)}_{Z(G_{ar}, G_{sr})} \right). \tag{31}$$

The CDF and PDF of the random variable  $Z(G_{ar}, G_{sr})$  can be calculated as follows:

- If  $\hat{P}_s\ell_{sr} \neq P_a\ell_{ar}$ , the CDF  $F_Z(z) = \Pr \{Z(G_{ar}, G_{sr}) \leq z\}$  can be derived as

$$\begin{aligned} F_Z(z) &= 1 - \frac{\hat{P}_s\ell_{sr}}{\hat{P}_s\ell_{sr} - P_a\ell_{ar}} \exp \left[ -\frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right) \eta \hat{P}_s\ell_{sr}} \right] \\ &\quad + \frac{P_a\ell_{ar}}{\hat{P}_s\ell_{sr} - P_a\ell_{ar}} \exp \left[ -\frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right) \eta P_a\ell_{ar}} \right]. \end{aligned} \tag{32}$$

The PDF  $f_Z(z) = \frac{dF_Z(z)}{dz}$  can be derived as

$$\begin{aligned} f_Z(z) &= \frac{1}{\eta \left(\frac{2\tau}{1-\tau} + \rho\right) \left(\hat{P}_s\ell_{sr} - P_a\ell_{ar}\right)} \\ &\quad \times \left\{ \exp \left[ -\frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right) \eta \hat{P}_s\ell_{sr}} \right] - \exp \left[ -\frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right) \eta P_a\ell_{ar}} \right] \right\}. \end{aligned} \tag{33}$$

- If  $\hat{P}_s\ell_{sr} = P_a\ell_{ar}$ , the CDF  $F_Z(z) = \Pr \{Z(G_{ar}, G_{sr}) \leq z\}$  can be derived as

$$F_Z(z) = 1 - \exp \left[ -\frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right)\eta\hat{P}_s\ell_{sr}} \right] - \frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right)\eta\hat{P}_s\ell_{sr}} \exp \left[ -\frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right)\eta P_a\ell_{ar}} \right]. \tag{34}$$

The PDF  $f_Z(z) = \frac{dF_Z(z)}{dz}$  can be derived as

$$f_Z(z) = \frac{1}{\left(\frac{2\tau}{1-\tau} + \rho\right)\eta\hat{P}_s\ell_{sr}} \left\{ \exp \left[ -\frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right)\eta\hat{P}_s\ell_{sr}} \right] + \left[ \frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right)\eta P_a\ell_{ar}} - 1 \right] \exp \left[ -\frac{z}{\left(\frac{2\tau}{1-\tau} + \rho\right)\eta P_a\ell_{ar}} \right] \right\}. \tag{35}$$

**4.2.1 Case B.1**

If  $\frac{I_0}{G_{rp}\ell_{rp}} < \hat{P}_r$  and  $\frac{I_0}{G_{rp}\ell_{rp}} < Z(G_{ar}, G_{sr})$ , we have  $P_r = \frac{I_0}{G_{rp}\ell_{rp}}$ . We have two conditions to represent the occurrence of this case:

$$\begin{aligned} \text{Case B.1.1: } P_s &= \hat{P}_s, P_r = \frac{I_0}{G_{rp}\ell_{rp}}, \\ G_{rp} &> \frac{I_0}{\hat{P}_r\ell_{rp}}, Z(G_{ar}, G_{sr}) > \hat{P}_r, \end{aligned} \tag{36}$$

$$\begin{aligned} \text{Case B.1.2: } P_s &= \hat{P}_s, P_r = \frac{I_0}{G_{rp}\ell_{rp}}, \\ G_{rp} &> \frac{I_0}{Z(G_{ar}, G_{sr})\ell_{rp}}, Z(G_{ar}, G_{sr}) < \hat{P}_r. \end{aligned} \tag{37}$$

Based on Case B.1.1, the probability  $P_{\text{outp}}^{\text{b11}}$  in (19) can be derived as

$$P_{\text{outp}}^{\text{b11}} = \left[ 1 - F_Z(\hat{P}_r) \right] \exp \left( -\frac{I_0}{\hat{P}_r\ell_{rp}} \right) \left\{ 1 - \exp \left[ -\frac{\xi_p(I_0 + N_0)}{P_a\ell_{ap}} \right] \right\}, \tag{38}$$

where  $F_Z(\hat{P}_r)$  can be obtained according to (32) and (34) with  $\hat{P}_s\ell_{sr} \neq P_a\ell_{ar}$  or  $\hat{P}_s\ell_{sr} = P_a\ell_{ar}$ , respectively.

Based on Case B.1.2, the probability  $P_{\text{outp}}^{\text{b12}}$  in (19) can be derived as

$$P_{\text{outp}}^{\text{b12}} = \left\{ 1 - \exp \left[ -\frac{\xi_p(I_0 + N_0)}{P_a\ell_{ap}} \right] \right\} \int_0^{\hat{P}_r} \exp \left( -\frac{I_0}{z\ell_{rp}} \right) f_Z(z) dz, \tag{39}$$

where the integral can be numerically calculated with  $f_Z(z)$  given in (33) and (35) with  $\hat{P}_s \ell_{sr} \neq P_a \ell_{ar}$  and  $\hat{P}_s \ell_{sr} = P_a \ell_{ar}$ , respectively.

**4.2.2 Case B.2**

If  $\hat{P}_r < \frac{I_0}{G_{rp} \ell_{rp}}$  and  $\hat{P}_r < Z(G_{ar}, G_{sr})$ , we have  $P_r = \hat{P}_r$ . We have one condition to represent the occurrence of this case:

$$\begin{aligned} \text{Case B.2: } P_s = \hat{P}_s, P_r = \hat{P}_r, \\ G_{rp} < \frac{I_0}{\hat{P}_r \ell_{rp}}, Z(G_{ar}, G_{sr}) > \hat{P}_r. \end{aligned} \tag{40}$$

Based on Case B.2, the probability  $P_{\text{outp}}^{\text{b2}}$  in (19) can be derived as

$$\begin{aligned} P_{\text{outp}}^{\text{b2}} = & \left[ 1 - F_Z(\hat{P}_r) \right] \left\{ 1 - \exp\left(-\frac{I_0}{\hat{P}_r \ell_{rp}}\right) - \frac{P_a \ell_{ap}}{P_a \ell_{ap} + \xi_p \hat{P}_r \ell_{rp}} \right. \\ & \left. \times \exp\left(-\frac{\xi_p N_0}{P_a \ell_{ap}}\right) \left[ 1 - \exp\left(-\frac{I_0}{\hat{P}_r \ell_{rp}} - \frac{\xi_p I_0}{P_a \ell_{ap}}\right) \right] \right\}, \end{aligned} \tag{41}$$

where  $F_Z(\hat{P}_r)$  can be obtained in (32) and (34) with  $\hat{P}_s \ell_{sr} \neq P_a \ell_{ar}$  and  $\hat{P}_s \ell_{sr} = P_a \ell_{ar}$ , respectively.

**4.2.3 Case B.3**

If  $Z(G_{ar}, G_{sr}) < \frac{I_0}{G_{rp} \ell_{rp}}$  and  $Z(G_{ar}, G_{sr}) < \hat{P}_r$ , we have  $P_r = Z(G_{ar}, G_{sr})$ . We have one condition to represent the occurrence of this case:

$$\begin{aligned} \text{Case B.3: } P_s = \hat{P}_s, P_r = Z(G_{ar}, G_{sr}), \\ G_{rp} < \frac{I_0}{Z(G_{ar}, G_{sr}) \ell_{rp}}, Z(G_{ar}, G_{sr}) < \hat{P}_r. \end{aligned} \tag{42}$$

Based on Case B.3, the probability  $P_{\text{outp}}^{\text{b3}}$  in (19) can be derived as

$$\begin{aligned} P_{\text{outp}}^{\text{b3}} = & \int_0^{\hat{P}_r} \left\{ 1 - \exp\left(-\frac{I_0}{z \ell_{rp}}\right) - \frac{P_a \ell_{ap}}{P_a \ell_{ap} + \xi_p \ell_{rp} z} \exp\left(-\frac{\xi_p N_0}{P_a \ell_{ap}}\right) \right. \\ & \left. \times \left[ 1 - \exp\left(-\frac{I_0}{z \ell_{rp}} - \frac{\xi_p I_0}{P_a \ell_{ap}}\right) \right] \right\} f_Z(z) dz, \end{aligned} \tag{43}$$

where the integral can be numerically calculated with  $f_Z(z)$  given in (33) and (35) with  $\hat{P}_s \ell_{sr} \neq P_a \ell_{ar}$  and  $\hat{P}_s \ell_{sr} = P_a \ell_{ar}$ , respectively.

Finally, we can numerically calculate the probability given in (19) by substituting the probabilities of (25), (26), (28), (30), (38), (39), (41), and (43), respectively.

### 5 Outage probability of the secondary system

The average outage probability can be approximated as

$$\begin{aligned}
 P_{\text{outs}} = & (1 - \Pr\{\mathcal{D}\}) + \underbrace{\Pr\{\mathcal{D}_a\} \Pr\left\{C_{\text{rd}}^{\text{III}} < \nu_s \mid P_s = \frac{I_0}{G_{\text{sp}}\ell_{\text{sp}}}\right\}}_{P_{\text{outs}}^{\text{a11}} + P_{\text{outs}}^{\text{a12}} + P_{\text{outs}}^{\text{a2}} + P_{\text{outs}}^{\text{a3}}} \\
 & + \underbrace{\Pr\{\mathcal{D}_b\} \Pr\left\{C_{\text{rd}}^{\text{III}} < \nu_s \mid P_s = \hat{P}_s\right\}}_{P_{\text{outs}}^{\text{b11}} + P_{\text{outs}}^{\text{b12}} + P_{\text{outs}}^{\text{b2}} + P_{\text{outs}}^{\text{b3}}}.
 \end{aligned} \tag{44}$$

The first term represents the probability of SR erroneously decoding data from SS in Phase II. The second and third terms represent the probabilities of SD erroneously decoding data from SR in Phase III, when the transmit power of SS in Phase II is  $\frac{I_0}{G_{\text{sp}}\ell_{\text{sp}}}$  or  $\hat{P}_s$ , respectively. The probabilities of events  $\mathcal{D}_a$  and  $\mathcal{D}_b$  are given in (13) and (14), respectively. In order to simplify the analysis, we analyze the occurrence probabilities of events  $\mathcal{D}_a$  and  $\mathcal{D}_b$  separately and omit their dependence with the CR, of which the accuracy will be verified through simulations. The eight probabilities correspond to the aforementioned eight cases, i.e., Case A.1.1, Case A.1.2, Case A.2, Case A.3, Case B.1.1, Case B.1.2, Case B.2, and Case B.3.

#### 5.1 Outage probabilities in Case A

Based on Case A.1.1 with conditions (23), we can derive  $P_{\text{outs}}^{\text{a11}}$  in (44) as

$$\begin{aligned}
 P_{\text{outs}}^{\text{a11}} = & \left[1 - F_Y(\hat{P}_r)\right] \exp\left(-\frac{I_0}{\hat{P}_r\ell_{\text{rp}}}\right) \left\{1 - \exp\left(-\frac{\xi_s N_0}{\hat{P}_r\ell_{\text{rd}}}\right)\right. \\
 & \times \int_0^\infty \frac{I_0\ell_{\text{rd}}}{I_0\ell_{\text{rd}} + \xi_s(N_0 + P_a G_{\text{ad}}\ell_{\text{ad}})\ell_{\text{rp}}} \\
 & \left. \times \exp\left[-\left(1 + \frac{\xi_s P_a \ell_{\text{ad}}}{\hat{P}_r\ell_{\text{rd}}}\right)G_{\text{ad}}\right] dG_{\text{ad}}\right\},
 \end{aligned} \tag{45}$$

where  $F_Y(\hat{P}_r)$  can be calculated according to (21) by replacing  $y$  with  $\hat{P}_r$ .

Based on Case A.1.2 with conditions (24), we can derive  $P_{\text{outs}}^{\text{a12}}$  in (44) as

$$\begin{aligned}
 P_{\text{outs}}^{\text{a12}} = & \int_0^{\hat{P}_r} \exp\left(-\frac{I_0}{y\ell_{\text{rp}}}\right) \left\{1 - \exp\left(-\frac{\xi_s N_0}{y\ell_{\text{rd}}}\right)\right. \\
 & \times \int_0^\infty \frac{I_0\ell_{\text{rd}}}{I_0\ell_{\text{rd}} + \xi_s(N_0 + P_a G_{\text{ad}}\ell_{\text{ad}})\ell_{\text{rp}}} \\
 & \left. \times \exp\left[-\left(1 + \frac{\xi_s P_a \ell_{\text{ad}}}{y\ell_{\text{rd}}}\right)G_{\text{ad}}\right] dG_{\text{ad}}\right\} f_Y(y) dy,
 \end{aligned} \tag{46}$$

where  $f_Y(y)$  is given in (22).

Based on Case A.2 with conditions (27), we can derive  $P_{\text{outs}}^{\text{a2}}$  in (44) as

$$\begin{aligned}
 P_{\text{outs}}^{\text{a2}} &= \left[ 1 - F_Y(\hat{P}_r) \right] \left[ 1 - \exp \left( -\frac{I_0}{\hat{P}_r \ell_{\text{rp}}} \right) \right] \\
 &\times \left[ 1 - \frac{\hat{P}_r \ell_{\text{rd}}}{\hat{P}_r \ell_{\text{rd}} + \xi_s P_a \ell_{\text{ad}}} \exp \left( -\frac{\xi_s N_0}{\hat{P}_r \ell_{\text{rd}}} \right) \right],
 \end{aligned} \tag{47}$$

where  $F_Y(\hat{P}_r)$  can be calculated according to (21) by replacing  $y$  with  $\hat{P}_r$ .

Based on Case A.3 with conditions (29), we can derive  $P_{\text{outs}}^{\text{a3}}$  in (44) as

$$\begin{aligned}
 P_{\text{outs}}^{\text{a3}} &= \int_0^{\hat{P}_r} \left[ 1 - \frac{y \ell_{\text{rd}}}{y \ell_{\text{rd}} + \xi_s P_a \ell_{\text{ad}}} \exp \left( -\frac{\xi_s N_0}{y \ell_{\text{rd}}} \right) \right] \\
 &\times \left[ 1 - \exp \left( -\frac{I_0}{y \ell_{\text{rp}}} \right) \right] f_Y(y) dy,
 \end{aligned} \tag{48}$$

where  $f_Y(y)$  is given in (22).

### 5.2 Outage probabilities in Case B

Based on Case B.1.1 with conditions (36), we can derive  $P_{\text{outs}}^{\text{b11}}$  in (44) as

$$\begin{aligned}
 P_{\text{outs}}^{\text{b11}} &= \left\{ \exp \left( -\frac{I_0}{\hat{P}_r \ell_{\text{rp}}} \right) - \int_{\frac{I_0}{\hat{P}_r \ell_{\text{rp}}}}^{\infty} \frac{I_0 \ell_{\text{rd}}}{I_0 \ell_{\text{rd}} + \xi_s P_a \ell_{\text{ad}} G_{\text{rp}} \ell_{\text{rp}}} \right. \\
 &\times \left. \exp \left[ -\left( 1 + \frac{\xi_s N_0 \ell_{\text{rp}}}{I_0 \ell_{\text{rd}}} \right) G_{\text{rp}} \right] dG_{\text{rp}} \right\} \left[ 1 - F_Z(\hat{P}_r) \right],
 \end{aligned} \tag{49}$$

where  $F_Z(\hat{P}_r)$  can be obtained according to (32) or (34) with  $\hat{P}_s \ell_{\text{sr}} \neq P_a \ell_{\text{ar}}$  or  $\hat{P}_s \ell_{\text{sr}} = P_a \ell_{\text{ar}}$ , respectively.

Based on Case B.1.2 with conditions (37), we can derive  $P_{\text{outs}}^{\text{b12}}$  in (44) as

$$\begin{aligned}
 P_{\text{outs}}^{\text{b12}} &= \int_0^{\hat{P}_r} \left\{ \exp \left( -\frac{I_0}{z \ell_{\text{rp}}} \right) - \int_{\frac{I_0}{z \ell_{\text{rp}}}}^{\infty} \frac{I_0 \ell_{\text{rd}}}{I_0 \ell_{\text{rd}} + \xi_s P_a \ell_{\text{ad}} G_{\text{rp}} \ell_{\text{rp}}} \right. \\
 &\times \left. \exp \left[ -\left( 1 + \frac{\xi_s N_0 \ell_{\text{rp}}}{I_0 \ell_{\text{rd}}} \right) G_{\text{rp}} \right] dG_{\text{rp}} \right\} f_Z(z) dz,
 \end{aligned} \tag{50}$$

where  $f_Z(z)$  is given in (33) and (35) with  $\hat{P}_s \ell_{\text{sr}} \neq P_a \ell_{\text{ar}}$  and  $\hat{P}_s \ell_{\text{sr}} = P_a \ell_{\text{ar}}$ , respectively.

Based on Case B.2 with conditions (40), we can derive  $P_{\text{outs}}^{\text{b2}}$  in (44) as

$$\begin{aligned}
 P_{\text{outs}}^{\text{b2}} &= \left[ 1 - F_Z(\hat{P}_r) \right] \left[ 1 - \exp \left( -\frac{I_0}{\hat{P}_r \ell_{\text{rp}}} \right) \right] \\
 &\times \left[ 1 - \frac{\hat{P}_r \ell_{\text{rd}}}{\hat{P}_r \ell_{\text{rd}} + \xi_s P_a \ell_{\text{ad}}} \exp \left( -\frac{\xi_s N_0}{\hat{P}_r \ell_{\text{rd}}} \right) \right],
 \end{aligned} \tag{51}$$

where  $F_Z(\hat{P}_r)$  is given in (32) and (34) with  $\hat{P}_s \ell_{\text{sr}} \neq P_a \ell_{\text{ar}}$  and  $\hat{P}_s \ell_{\text{sr}} = P_a \ell_{\text{ar}}$ , respectively.

Based on Case B.3 with conditions (42), we can derive  $P_{\text{outs}}^{\text{b3}}$  in (44) as

$$P_{\text{outs}}^{\text{b3}} = \int_0^{\hat{P}_r} \left[ 1 - \exp\left(-\frac{I_0}{z l_{rp}}\right) \right] \times \left[ 1 - \frac{z l_{rd}}{z l_{rd} + \xi_s P_a l_{ad}} \exp\left(-\frac{\xi_s N_0}{z l_{rd}}\right) \right] f_Z(z) dz, \tag{52}$$

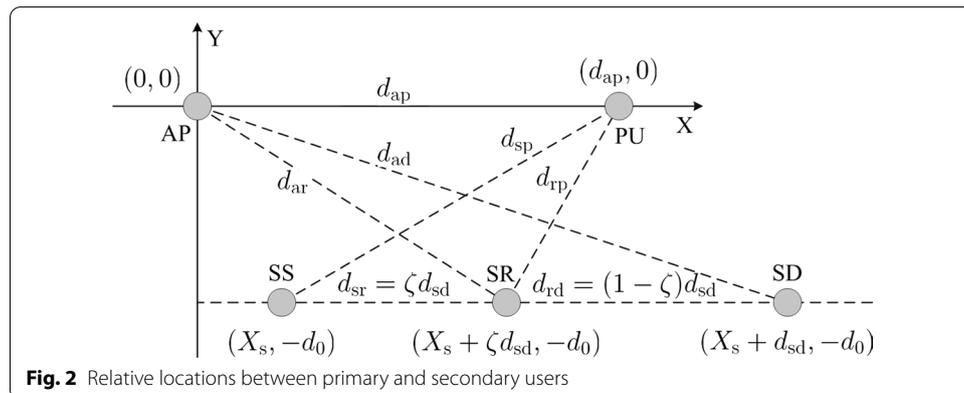
where  $f_Z(z)$  is given in (33) and (35) with  $\hat{P}_s l_{sr} \neq P_a l_{ar}$  and  $\hat{P}_s l_{sr} = P_a l_{ar}$ , respectively.

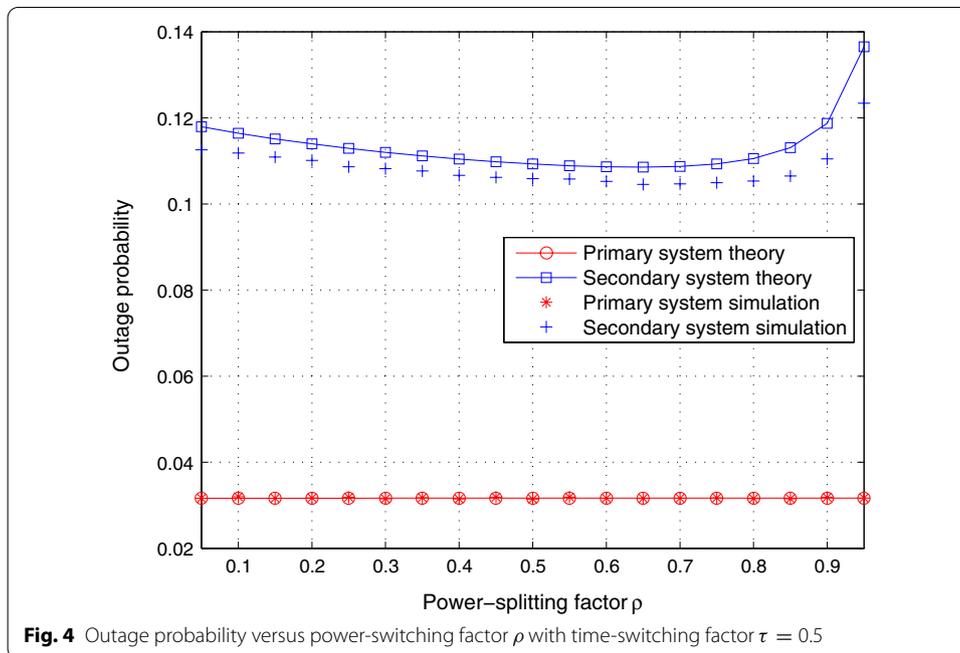
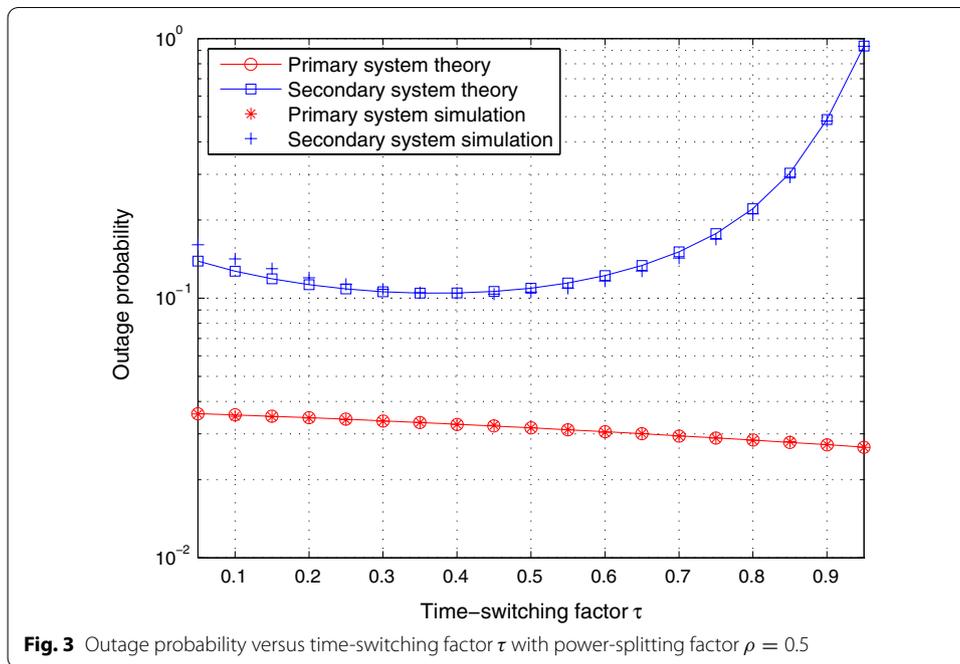
Finally, we can calculate the outage probability (44) by substituting the probabilities of (45), (46), (47), (48), (49), (50), (51), and (52), respectively.

### 6 Results and discussion

In the simulations, as shown in Fig. 2, AP and PU are located at the points of (0, 0) and  $(d_{ap}, 0)$ , respectively. The two lines SS→SR→SD and AP→PU are assumed to be parallel with a vertical distance  $d_0$ . SS, SR, and SD are located at the positions of  $(X_s, -d_0)$ ,  $(X_s + \zeta d_{sd}, -d_0)$ , and  $(X_s + d_{sd}, -d_0)$ , respectively, where  $\zeta \in (0, 1)$  represents the relative location of SR on the line between SS and SD, i.e.,  $d_{sr} = \zeta d_{sd}$  and  $d_{rd} = (1 - \zeta)d_{sd}$ . The X coordinate of SS that is  $X_s$  may be a positive or negative value. Therefore, the distances between different nodes can be calculated as  $d_{ar} = \sqrt{(X_s + \zeta d_{sd})^2 + d_0^2}$ ,  $d_{ad} = \sqrt{(X_s + d_{sd})^2 + d_0^2}$ ,  $d_{sp} = \sqrt{(X_s - d_{ap})^2 + d_0^2}$ , and  $d_{rp} = \sqrt{(X_s + \zeta d_{sd} - d_{ap})^2 + d_0^2}$ . Similarly as [20, 21], unless specified otherwise, the parameters are set as  $\eta = 0.8$ ,  $\alpha = 3$ ,  $P_a = 30$  dBm,  $\hat{P}_s = 30$  dBm,  $\hat{P}_r = 20$  dBm,  $I_0 = 0$  dBm,  $d_{ap} = 4$  m,  $d_{sd} = 4$  m,  $\zeta = 0.5$ ,  $d_0 = 8$  m,  $X_s = 0$ ,  $N_0 = 0.001$  W,  $\sigma_{na}^2 = \sigma_{nc}^2 = N_0/2$ ,  $v_p = 0.5$  bits/s/Hz, and  $v_s = 0.1$  bits/s/Hz.

We present two benchmark schemes for the comparison, which are TS-EH and PS-EH based CR. For the TS-EH based CR, each time block is divided into three phases, wherein the WPT is performed in the first phase with duration  $\tau$ , and the CR is performed in the following two phases with each phase lasting for  $\frac{1-\tau}{2}$  time. For the PS-EH based CR, each time block is divided into two equal-length phases. In the first phase, SS transmits the secondary data and AP transmits the primary data simultaneously. SR splits  $\rho$  fraction of the received composite signal for the EH, and the remaining part is used for the data decoding. When SR correctly decodes the secondary data, it will forward the data to SD in the second phase. For the TS-EH and PS-EH based schemes, the





outage probabilities of primary and secondary systems can be calculated as  $P_{\text{outp}}$  (15) and  $P_{\text{outs}}$  (44) by letting  $\rho = 0$  and  $\tau = 0$ , respectively.

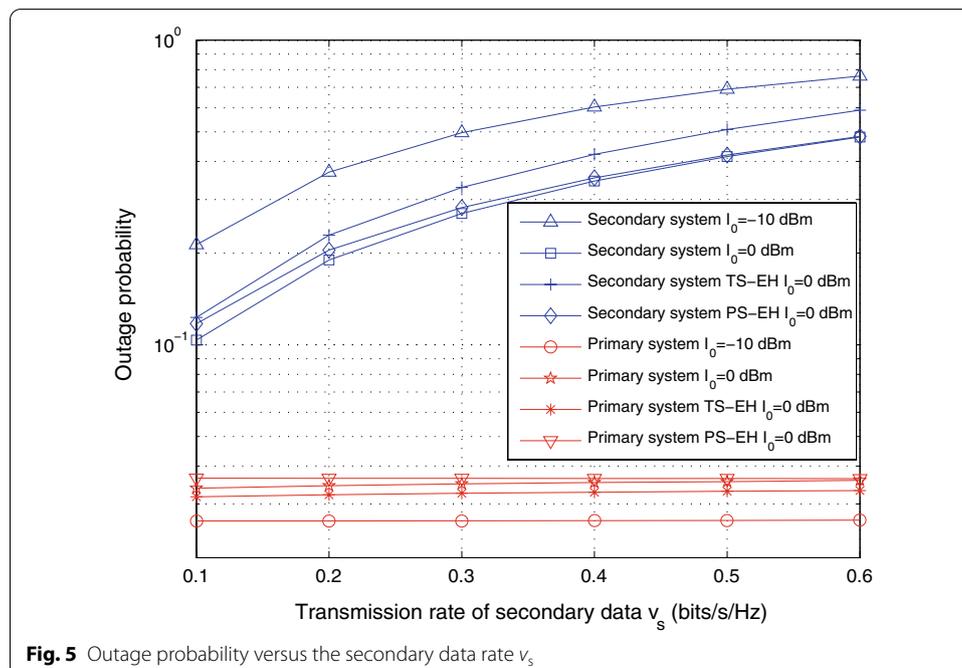
### 6.1 Impacts of TS factor and PS factor

Figure 3 shows the outage performance versus the TS factor. On one hand, with the increase of  $\tau$ , more energy can be harvested by SR, so SR could forward the secondary

data with higher power, which is helpful to decrease the outage probability. But, on the other hand, with the increase of  $\tau$ , less time is remained for the CR, which means the channel achievable rate gets smaller, causing higher outage probability. As a tradeoff of these two facts, the outage probability of the secondary system decreases first and then turns larger. Since PU can cancel the energy signal for the data decoding, with the increase of  $\tau$ , the duration of interference-free transmission gets longer, so the outage probability of the primary system gets smaller. Theoretical results and simulation results match well for the primary system, but not very well for the secondary system.

Figure 4 shows the outage performance versus the PS factor. On one hand, with the increase of  $\rho$ , more energy can be harvested by SR, so SR could forward the secondary data with higher power, which is beneficial to the outage performance. But, on the other hand, with the increase of  $\rho$ , less signal power is remained for the data decoding at SR in Phase II, which degrades the CR performance. As a tradeoff of these two facts, the outage probability of the secondary system decreases firsts and then increases. With the increase of  $\rho$ , the outage performance of the primary system is almost invariant. Theoretical results and simulation results match well for the primary system, but not good for the secondary system. because the approximate outage probability is derived in (44) by omitting the dependence of Phase-II-SR-decoding with the Phase-I-SR-EH and the Phase-III-SR-relaying.

As shown in Figs. 3 and 4, there exist optimal  $\tau$  and  $\rho$  that can minimize the outage probability of the secondary system. In the following simulations, we will jointly determine the optimal  $\tau$  and  $\rho$  by using the numerical search method.

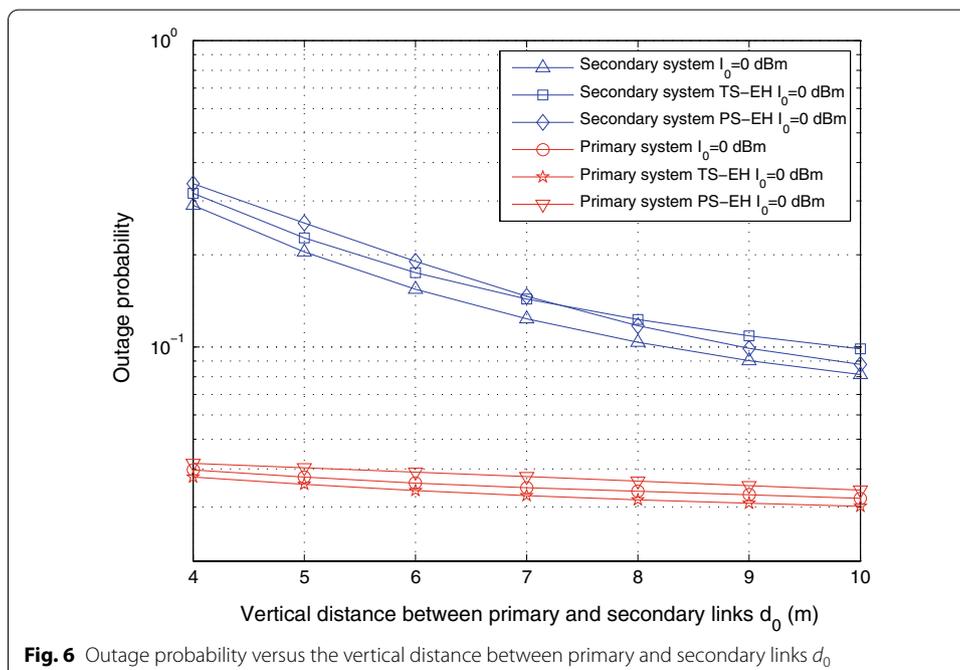


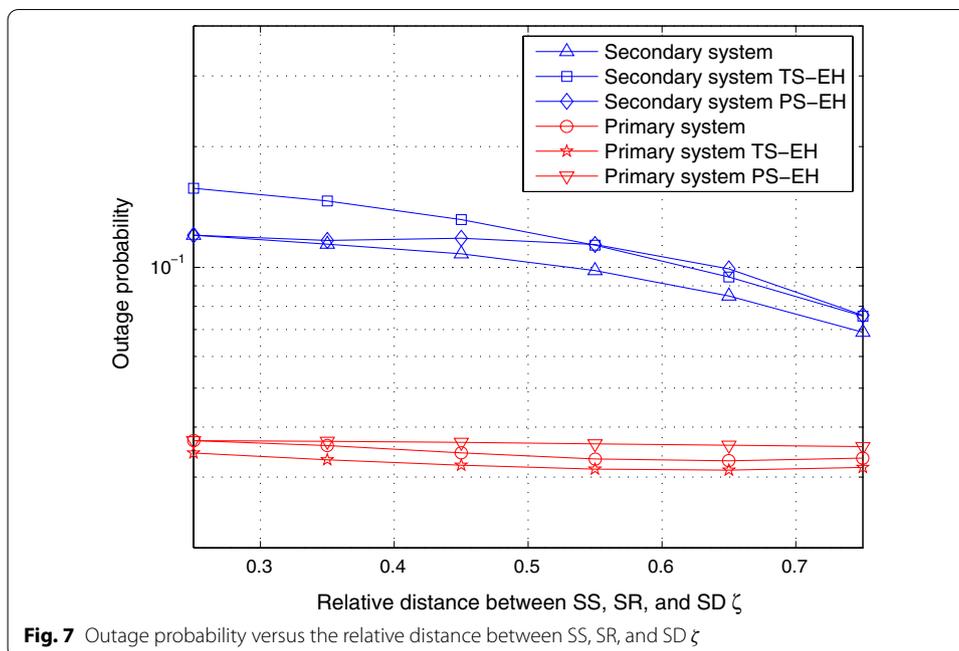
### 6.2 Outage probability with other parameters

Figure 5 shows the outage performance versus the transmission rate of secondary data. Given an interference threshold, with the increase of  $\nu_s$ , it becomes more difficult for SR to decode the secondary data. However, if SR succeeds in decoding the secondary data in Phase II and forwards it to SD in Phase III, the higher rate makes the decoding at SD more difficult. As a result, the outage probability of secondary system gets larger. For the primary system, given an interference threshold, with the increase of  $\nu_s$ , the outage probability changes slightly. When the interference threshold gets larger from  $-10$  to  $0$  dBm, the secondary outage probability gets smaller, as higher power can be allowed for the CR, while the primary outage probability gets larger, as more interference can be caused by the CR. For the secondary system, our proposed TS-PS-EH based CR performs better than the TS-EH and the PS-EH based schemes.

Figure 6 shows the outage performance versus the vertical distance between the two systems. With the increase of  $d_0$ , the outage probabilities of both systems decrease monotonically. This is because, the farther away the two systems departed, the weaker the mutual interference, which is helpful for the data decoding. For the secondary system, our proposed CR scheme achieves smaller outage probability than the TS-EH and the PS-EH based schemes, as the optimal  $\tau$  and  $\rho$  are jointly determined. For the primary system, the outage probability of our proposed scheme lies in-between the two benchmark schemes.

Figure 7 shows the outage performance versus the relative distance between SS, SR, and SD. With the increase of  $\zeta$ , SR departs farther away from SS, but closer to SD. On one hand, the longer distance from SS to SR harms the amount of harvested energy at SR, which lowers the possible transmit power of SR in Phase III. But, on the other hand, the weaker primary interference is caused to SR, which is helpful for the data decoding at SR in Phase II, and further the shorter distance from SR to SD can help improve the





data decoding performance at SD. As a compromise, for our parameter settings, the outage probability of secondary system decreases in the given range of  $\zeta$ . The outage probability of primary system decreases slightly, as the interference-free primary transmission is more likely in Phase III due to the more possible failure of SR decoding. Compared with the benchmark schemes, our proposed scheme achieves smaller outage probability for the secondary system, while lies in the middle area for the primary system.

### 7 Conclusion

We have proposed a DF based CR scheme for the underlay spectrum sharing scenario. Through jointly using TS and PS techniques, SR can harvest wireless energy from both SS and AP in both time and power domains. The transmit power SS is constrained by the interference threshold at PU and its allowable peak power. Apart from the above two constraints, the transmit power of SR is further constrained by the available energy. By considering all the possible transmit powers of SS and SR, we analyze the approximate outage probabilities for both primary and secondary systems by omitting the dependence between some variables. Numerical results verify the effectiveness of our analysis and show that our proposed scheme can achieve much better outage performance for the secondary system compared with the benchmark schemes.

If there are multiple PUs, the transmit powers of SS and SR should satisfy the interference constraints of all the PUs. If there are multiple SRs, the one that can correctly decode the secondary data from SS and achieves the best communication quality towards SD can be selected for the data relaying. A coordination protocol is required to select the best SR based on the amount of harvested energy, the data decoding status, and the channel state, etc.

### Abbreviations

AF: Amplify-and-forward; AP: Access point; AWGN: Additive white Gaussian noise; BS: Base station; CDF: Cumulative distribution function; CR: Cognitive relaying; CSI: Channel state information; DF: Decode-and-forward; EH: Energy harvesting; IoT: Internet of things; PDF: Probability density function; PS: Power-splitting; PSR: Power-splitting based relaying; PU: Primary user; RF: Radio frequency; RFID: Radio frequency identification; SD: Secondary destination; SR: Secondary relay; SS: Secondary source; SU: Secondary user; SIC: Successive interference cancellation; SINR: Signal-to-interference-plus-noise-ratio; SNR: Signal-to-noise-ratio; TS: Time-switching; TSR: Time-switching based relaying; WPT: Wireless power transfer.

### Authors' contributions

LZ drafted the paper, designed the transmission protocol, and participated in the outage performance analysis. CZ proposed the system model, analyzed the outage probabilities, and performed the simulations. All authors read and approved the final manuscript.

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### Availability of data and materials

The datasets used or analysed during the current study are available from the corresponding author on reasonable request.

### Declarations

#### Competing interests

The authors declare that they have no competing interests.

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